

三维移动目标无源定位算法

王小英¹, 江怡帆², 赵悦^{3,4}

(1. 华北电力大学数理学院, 北京 102206; 2. 多伦多大学统计系, 加拿大 M5S 2E8;
3. 北京应用物理与计算数学研究所, 北京 100094; 4. 东北师范大学数学与统计学院, 吉林长春 130024)

摘要: 无源定位已经成为雷达对抗中一项相当重要、不可或缺的技术。针对常态侦察时三维移动目标的无源定位问题, 考虑了基于到达方位信息的代数几何概率定位方法, 并给出了每一时刻目标位置误差随时间变化的关系, 最后通过数值模拟展示了所研究定位方法的精度。该算法基于运动学原理, 充分利用了辐射源信号测量值的时空变化量信息, 定位收敛速度快而且非常稳定, 具有很强的实用性。以期在实际应用中对近空间或邻近空间中高速移动目标进行三维快速定位, 更好地提供无源侦察中移动目标的方位情报信息。

关键词: 无源定位; 三维移动目标; 到达方位; 代数几何概率方法

中图分类号: TN974 文献标志码: A 文章编号: 1672-2337(2019)03-0280-05

An Algebraic Geometric Probabilistic Passive Location Method for Three-Dimensional Moving Target

WANG Xiaoying¹, JIANG Yifan², ZHAO Yue^{3,4}

(1. School of Mathematical Statistics, North China Electric Power University, Beijing 102206, China;
2. Department of Statistics, University of Toronto, Canada M5S 2E8;
3. Beijing Institute of Applied Physics and Computational Mathematics, Beijing 100094, China;
4. School of Mathematics and Statistics, Northeast Normal University, Changchun 130024, China)

Abstract: Passive positioning has become a very important and indispensable technology in radar confrontation. Aiming at the passive location of 3D moving targets in normal reconnaissance, an algebraic geometric probabilistic positioning method based on arrival position information is considered, and the relationship between target position error and time at each moment. Finally, numerical simulations show the accuracy of the proposed method. Based on the kinematics principle, the algorithm makes full use of the spatio-temporal change information of the measured value of radiation source signal, so that the positioning convergence speed is fast and very stable. This method could help three-dimensional rapid positioning of high-speed moving objects in near space and provide position information of moving targets in passive reconnaissance.

Key words: passive location; 3D moving target; arrival orientation; algebra geometry probability method

0 引言

辐射源无源定位技术^[1-3]是电子战的一个重要组成部分, 无论是传统的战略战术电子侦察, 还是电子干扰, 都离不开利用目标辐射源的信号或外辐射源的信号对目标进行定位。无源定位技术

能在自身不辐射的条件下, 隐蔽地确定辐射源的位置, 具有作用距离远、抗干扰能力强的特点, 对于提高系统在复杂电子战环境下的生存能力和作战效能具有十分重要的作用, 因此无源定位技术的研究越来越受到各国的重视。

无源定位的目标是得到辐射源的高精度二维或三维空间位置, 现阶段的多站无源定位技术主

$$E = (x_2 - x_3)(\tan^2 \gamma_1 - \tan^2 \gamma_2) - (x_2 - x_1)(\tan^2 \gamma_3 - \tan^2 \gamma_2)$$

计算可得

$$x_R = \frac{A + D \cdot (z_R - z_1)^2}{2C} \quad (6)$$

$$y_R = -\frac{B + E \cdot (z_R - z_1)^2}{2C} \quad (7)$$

$$(D^2 + E^2)(z_R - z_1)^4 + [(A - 2x_1C)^2 + (B + 2y_1C)^2] + [2(A - 2x_1C) + 2(B + 2y_1C)E - 4C^2 \tan^2 \gamma_1](z_R - z_1)^2 = 0 \quad (8)$$

在给出由侦察站的位置以及目标电磁波辐射到达角误差引起的辐射源位置协方差矩阵之前, 首先给出下面一个关于复合函数求协方差矩阵的链式法则。

引理 1 (三维复合函数协方差矩阵链式法则) 假设由零均值随机变量 X, Y, Z 组成的 3 个函数 f_1, f_2, f_3 , 其方差分别为 $\sigma_x^2, \sigma_y^2, \sigma_z^2$, 相关性为 $\rho_{XY}, \rho_{YZ}, \rho_{XZ}$, 那么 f_1, f_2, f_3 的协方差矩阵为

$$\mathbf{Cov}(f_1, f_2, f_3) = \mathbf{J}_{\frac{f_1 f_2 f_3}{xyz}} \cdot \mathbf{Cov}(X, Y, Z) \cdot \mathbf{J}_{\frac{f_1 f_2 f_3}{xyz}}^T \quad (9)$$

式中

$$\mathbf{Cov}(X, Y, Z) = \begin{pmatrix} \sigma_x^2 & \rho_{XY} \sigma_x \sigma_y & \rho_{XZ} \sigma_x \sigma_z \\ \rho_{XY} \sigma_x \sigma_y & \sigma_y^2 & \rho_{YZ} \sigma_y \sigma_z \\ \rho_{XZ} \sigma_x \sigma_z & \rho_{YZ} \sigma_y \sigma_z & \sigma_z^2 \end{pmatrix}$$

$$\mathbf{J}_{\frac{f_1 f_2 f_3}{xyz}} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix}$$

$$\mathbf{Cov}(f_1, f_2, f_3) =$$

$$\begin{pmatrix} \sigma_{f_1}^2 & \rho_{f_1 f_2} \sigma_{f_1} \sigma_{f_2} & \rho_{f_1 f_3} \sigma_{f_1} \sigma_{f_3} \\ \rho_{f_1 f_2} \sigma_{f_1} \sigma_{f_2} & \sigma_{f_2}^2 & \rho_{f_2 f_3} \sigma_{f_2} \sigma_{f_3} \\ \rho_{f_1 f_3} \sigma_{f_1} \sigma_{f_3} & \rho_{f_2 f_3} \sigma_{f_2} \sigma_{f_3} & \sigma_{f_3}^2 \end{pmatrix}$$

证根据全微分定义可得下面的近似等式 ($m, n = 1, 2, 3$):

$$\sum_{i=1}^N \langle f_{im} - \bar{f}_m, f_{in} - \bar{f}_n \rangle \approx \sum_{i=1}^N \left\langle \frac{\partial f_{im}}{\partial x} (x_i - \bar{x}) + \frac{\partial f_{im}}{\partial y} (y_i - \bar{y}) + \frac{\partial f_{im}}{\partial z} (z_i - \bar{z}), \frac{\partial f_{in}}{\partial x} (x_i - \bar{x}) + \frac{\partial f_{in}}{\partial y} (y_i - \bar{y}) + \frac{\partial f_{in}}{\partial z} (z_i - \bar{z}) \right\rangle$$

$$\begin{aligned} \frac{\partial f_{in}}{\partial z} (z_i - \bar{z}) \rangle &= \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial x} \sigma_x^2 + \\ & \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial y} \rho_{XY} \sigma_x \sigma_y + \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial z} \rho_{XZ} \sigma_x \sigma_z + \\ & \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial y} \sigma_y^2 + \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial z} \rho_{YZ} \sigma_y \sigma_z + \\ & \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial x} \rho_{XZ} \sigma_x \sigma_z + \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial z} \sigma_z^2 + \\ & \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial x} \rho_{YX} \sigma_y \sigma_x + \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial y} \rho_{YZ} \sigma_y \sigma_z \end{aligned} \quad (10)$$

另一方面, 经计算可得

$$\mathbf{J}_{\frac{f_1 f_2 f_3}{xyz}} \cdot \mathbf{Cov}(X, Y, Z) \cdot \mathbf{J}_{\frac{f_1 f_2 f_3}{xyz}}^T = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (11)$$

式中 ($m, n = 1, 2, 3$),

$$\begin{aligned} a_{mm} &= \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial x} \sigma_x^2 + \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial y} \rho_{XY} \sigma_x \sigma_y + \frac{\partial f_m}{\partial x} \frac{\partial f_n}{\partial z} \rho_{XZ} \sigma_x \sigma_z + \\ & \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial y} \sigma_y^2 + \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial z} \rho_{YZ} \sigma_y \sigma_z + \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial x} \rho_{XZ} \sigma_x \sigma_z + \\ & \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial z} \sigma_z^2 + \frac{\partial f_m}{\partial y} \frac{\partial f_n}{\partial x} \rho_{YX} \sigma_y \sigma_x + \frac{\partial f_m}{\partial z} \frac{\partial f_n}{\partial y} \rho_{YZ} \sigma_y \sigma_z \end{aligned}$$

比较式(10)和式(11)可以得到复合函数协方差矩阵的链式法则式(9)。

由上面的引理, 可以得到辐射源位置的协方差矩阵为

$$\begin{aligned} \mathbf{Cov}(x_R(t), y_R(t), z_R(t)) &= \\ & \mathbf{J}_{\frac{x_R y_R z_R}{x_i y_i z_i}}(t) \cdot \mathbf{Cov}(x_i(t), y_i(t), z_i(t)) \cdot \mathbf{J}_{\frac{x_R y_R z_R}{x_i y_i z_i}}^T(t) + \\ & \mathbf{J}_{\frac{x_R y_R z_R}{\gamma_1 \gamma_2 \gamma_3}}(t) \cdot \mathbf{Cov}(\gamma_1(t), \gamma_2(t), \gamma_3(t)) \cdot \mathbf{J}_{\frac{x_R y_R z_R}{\gamma_1 \gamma_2 \gamma_3}}^T(t) \end{aligned} \quad (12)$$

式中,

$$\mathbf{J}_{\frac{x_R y_R z_R}{\gamma_1 \gamma_2 \gamma_3}}(t) = \begin{pmatrix} \frac{\partial x_R}{\partial \gamma_1} & \frac{\partial x_R}{\partial \gamma_2} & \frac{\partial x_R}{\partial \gamma_3} \\ \frac{\partial y_R}{\partial \gamma_1} & \frac{\partial y_R}{\partial \gamma_2} & \frac{\partial y_R}{\partial \gamma_3} \\ \frac{\partial z_R}{\partial \gamma_1} & \frac{\partial z_R}{\partial \gamma_2} & \frac{\partial z_R}{\partial \gamma_3} \end{pmatrix}_{3 \times 3}$$

$$\mathbf{J}_{\frac{x_R y_R z_R}{x_i y_i z_i}}(t) =$$

$$\begin{pmatrix} \frac{\partial x_R}{\partial x_1} & \frac{\partial x_R}{\partial y_1} & \frac{\partial x_R}{\partial z_1} & \frac{\partial x_R}{\partial x_2} & \frac{\partial x_R}{\partial y_2} & \frac{\partial x_R}{\partial z_2} & \frac{\partial x_R}{\partial x_3} & \frac{\partial x_R}{\partial y_3} & \frac{\partial x_R}{\partial z_3} \\ \frac{\partial y_R}{\partial x_1} & \frac{\partial y_R}{\partial y_1} & \frac{\partial y_R}{\partial z_1} & \frac{\partial y_R}{\partial x_2} & \frac{\partial y_R}{\partial y_2} & \frac{\partial y_R}{\partial z_2} & \frac{\partial y_R}{\partial x_3} & \frac{\partial y_R}{\partial y_3} & \frac{\partial y_R}{\partial z_3} \\ \frac{\partial z_R}{\partial x_1} & \frac{\partial z_R}{\partial y_1} & \frac{\partial z_R}{\partial z_1} & \frac{\partial z_R}{\partial x_2} & \frac{\partial z_R}{\partial y_2} & \frac{\partial z_R}{\partial z_2} & \frac{\partial z_R}{\partial x_3} & \frac{\partial z_R}{\partial y_3} & \frac{\partial z_R}{\partial z_3} \end{pmatrix}_{3 \times 9}$$

2 算法及数值模拟

本节将对提出的三维空间移动目标无源定位算法进行数值验证。相关参数设置如表1所示,分别针对随机变量不同方差以及到达角不随时间变化的数值模拟结果如图2~4所示,目标位置以及椭球半径随方差变化的模拟数据如表2所示。

表1 数值模拟 AOA 方法参数设置

A 侦察站移动速度	$v_A=(2,2,2)$ m/s
B 侦察站移动速度	$v_B=(2,2,2)$ m/s
C 侦察站移动速度	$v_C=(2,2,2)$ m/s
A 侦察站起始位置	$(-5,0,0)$ km
B 侦察站起始位置	$(5,0,0)$ km
C 侦察站起始位置	$(0,5,0)$ km
误差随机变量样本点数	$N=10\ 000$
模拟总时间	$T=3$ s
时间步长	$\Delta t=0.5$ s

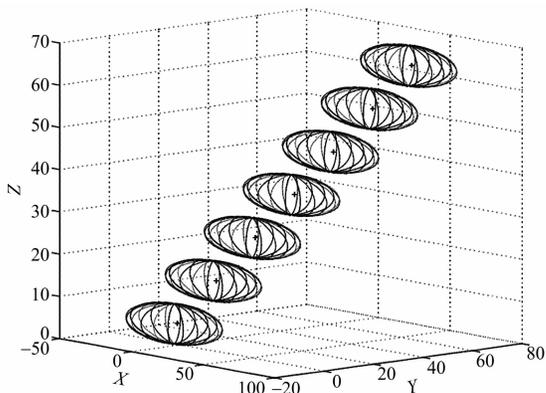


图2 三侦察站到达角不随时间变化情形
($\gamma_1=\pi/4, \gamma_2=\pi/6, \gamma_3=\pi/3$; 随机变量方差均为1)

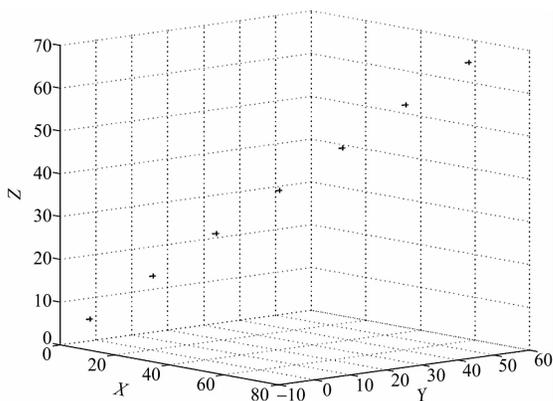


图3 三侦察站到达角不随时间变化情形
($\gamma_1=\pi/4, \gamma_2=\pi/6, \gamma_3=\pi/3$; 随机变量方差均为0.1)

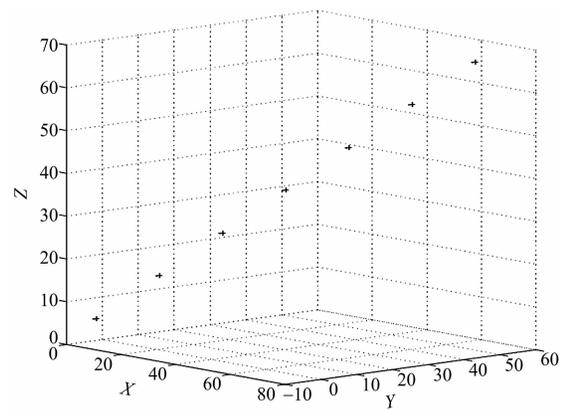


图4 三侦察站到达角不随时间变化情形
($\gamma_1=\pi/4, \gamma_2=\pi/6, \gamma_3=\pi/3$; 随机变量方差均为0.01)

表2 目标位置和椭球半径随方差变化结果
(AOA 不随时间变化)

方差 Var	目标位置 (x_R, y_R, z_R)/km	椭球半径 a, b, c /km
1	(0.400 0, -2.800 0, 5.216 6)	$a=9.678\ 9, b=8.330\ 5, c=4.664\ 6$
	(10.400 0, 7.200 0, 15.216 6)	$a=9.367\ 8, b=8.333\ 3, c=4.661\ 1$
	(20.400 0, 17.200 0, 25.216 6)	$a=9.679\ 0, b=8.331\ 2, c=4.656\ 6$
0.1	(0.400 0, -2.800 0, 5.216 6)	$a=3.053\ 2, b=0.835\ 2, c=0.466\ 4$
	(10.400 0, 7.200 0, 15.216 6)	$a=9.052\ 8, b=0.829\ 2, c=0.466\ 4$
	(20.400 0, 17.200 0, 25.216 6)	$a=9.052\ 5, b=0.834\ 9, c=0.466\ 0$
0.01	(0.400 0, -2.800 0, 5.216 6)	$a=0.304\ 1, b=0.083\ 7, c=0.046\ 9$
	(10.400 0, 7.200 0, 15.216 6)	$a=0.304\ 1, b=0.083\ 7, c=0.046\ 9$
	(20.400 0, 17.200 0, 25.216 6)	$a=0.304\ 1, b=0.083\ 7, c=0.046\ 9$

从表2和图2~4中可以发现,随着随机变量方差的减小,目标位置估计区域椭球的半径也在减小,即定位精度越来越高,这为实际中三维移动目标快速并精确定位提供了依据。

另外对于到达角随时间变化的数值模拟结果如图5~7所示,目标位置以及椭球半径随方差变化的模拟数据如表3所示。

表3 目标位置和椭球半径随方差变化结果
(AOA 随时间变化)

方差 Var	目标位置 (x_R, y_R, z_R)/km	椭球半径 a, b, c /km
1	(0, -3.75, 5.077 5)	$a=9.723\ 6, b=8.379\ 7, c=4.670\ 8$
	(1, -2.75, 6.077 5)	$a=9.677\ 2, b=8.421\ 7, c=4.652\ 8$
	(2, -1.75, 7.007 5)	$a=9.762\ 8, b=8.303\ 8, c=4.647\ 9$
0.1	(0, -3.75, 5.077 5)	$a=3.059\ 1, b=0.846\ 3, c=0.468\ 0$
	(1, -2.75, 6.077 5)	$a=3.057\ 5, b=0.837\ 7, c=0.467\ 9$
	(2, -1.75, 7.007 5)	$a=3.059\ 5, b=0.837\ 8, c=0.463\ 0$
0.01	(0, -3.75, 5.077 5)	$a=0.306\ 9, b=0.084\ 3, c=0.046\ 9$
	(1, -2.75, 6.077 5)	$a=0.306\ 9, b=0.084\ 3, c=0.046\ 9$
	(2, -1.75, 7.007 5)	$a=0.306\ 9, b=0.084\ 3, c=0.046\ 9$

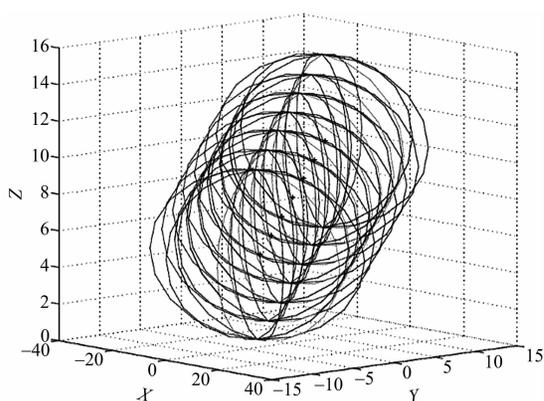


图 5 三侦察站到达角随时间变化情形
(随机变量方差均为 1)

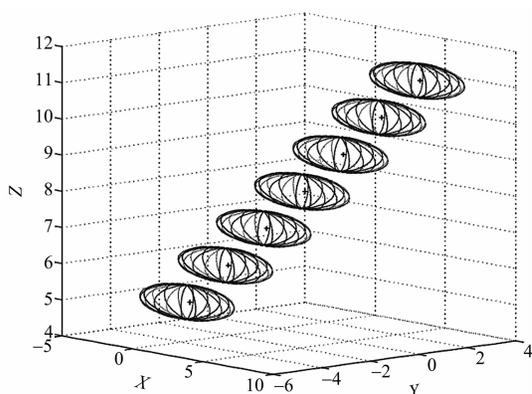


图 6 三侦察站到达角随时间变化情形
(随机变量方差均为 0.1)

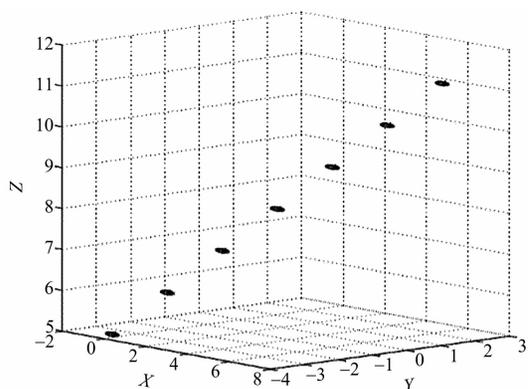


图 7 三侦察站到达角随时间变化情形
(随机变量方差均为 0.01)

3 结束语

本文提出了基于到达方位信息的常态侦察时三维移动目标无源定位代数几何概率方法,并通过数值模拟展示了所研究定位方法的精度。本文所提的方法可为近空间或邻近空间中高速移动目

标的三维快速定位提供一定的借鉴意义。为进一步提高定位精度,下一步可考虑基于到达时间差的方法和基于到达时间差/到达频率差的方法对三维移动目标进行定位。

参考文献:

- [1] 孙仲康,周一宇,何黎星. 单多基地有源无源定位技术[M]. 北京:国防工业出版社,1996:24-116.
- [2] 刘霞,龙飞,张延升. 雷达机动目标跟踪无源定位优化研究[J]. 计算机仿真,2016,33(3):1-4.
- [3] 牟之英,刘博. 多平台对多目标无源融合定位方法[J]. 探测与控制学报,2017,39(1):58-65.
- [4] 曹志亮,汪晋,任芹,等. 基于纯角度信息的单站无源定位算法[J]. 电子技术与软件工程,2017(10):118-120.
- [5] 张卓然,叶广强,刘华伟. 一种机载单站对固定目标的无源定位方法[J]. 计算机仿真,2015,32(2):92-95.
- [6] HO K C, CHAN Y T. Solution and Performance Analysis of Geolocation by TDOA[J]. IEEE Trans on Aerospace and Electronic Systems, 1993, 29(4): 1311-1322.
- [7] HO K C, XU Wenwei. An Accurate Algebraic Solution for Moving Source Location Using TDOA and FDOA Measurements[J]. IEEE Trans on Signal Processing, 2004, 52(9): 2453-2463.
- [8] 刘洋,杨乐,郭福成,等. 基于定位误差修正的运动目标 TDOA/FDOA 无源定位方法[J]. 航空学报,2015, 36(5):1617-1626.
- [9] 方锋,郭徽东. 基于到达时间和方位角的固定单站无源定位仿真分析[J]. 舰船电子对抗,2016,39(2):10-13.
- [10] 李宗华,冯道旺,周一宇,等. 估计目标速度矢量对运动辐射源的固定单站无源定位算法[J]. 电子学报,2004,32(6):953-956.
- [11] 李宗华,冯道旺,周一宇,等. 固定单站对三维运动辐射源无源定位算法及可观测性分析[J]. 信号处理,2004,20(2):117-121.

作者简介:



王小英 女,1978年生,河南人,博士,副教授,1998—2005年就读于东北师范大学数学与统计学院,2005—2009年获新加坡国立大学统计与应用概率系博士学位,2010年3月至今华北电力大学副教授,主要研究方向为大维随机矩阵、大维数据分析与统计计算,现负责国家自然科学基金和青年基金两项。

负责国家自然科学基金和青年基金两项。