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Global stability analysis in Hopfield neural networks

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Abstract: The existence and uniqueness of the equilibrium and the global attractivity of Hopfield neural network models are investigated. Instead of assuming the boundedness, monotonicity and differentiability of the activation functions and by using M-matrix theory, Lyapunov functions are constructed and employed to establish sufficient conditions for global asymptotic stability.

Key words: neural network; global asymptotic stability; M-matrix

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Hopfield 神经网络系统的全局稳定性分析

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摘要：研究一类 Hopfield 神经网络系统的平衡状态的存在性、唯一性与全局稳定性，这类系统放弃了以前对激励函数的有界性、单调性和可微性要求.利用 M 矩阵理论，通过构造适当的 Lyapunov 函数，得到了系统全局渐近稳定的充分条件.

关键词：神经网络；全局稳定性；M-矩阵

1 Introduction

There has recently been increasing interest in the potential applications of the dynamics of artificial neural networks in signal and image processing. Among the most popular models in the literature of artificial neural networks is Hopfield-type neural network described by the ordinary differential equations

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = -Bu(t) + Ag(u(t)) + J, \qquad (1)$$

where $u = (u_1, \cdots, u_n)^{\mathrm{T}}$, $B = \mathrm{diag}\,(b_1, \cdots, b_n) > 0$, the connection matrix $A = (a_{ij})_{n\times n}$, activation functions

$$g(u) = (g_1(u_1), \cdots, g_n(u_n))^{\mathrm{T}},$$
$$J = (J_1, \cdots, J_n)^{\mathrm{T}}.$$

Hopfield-type neural network (1) and their various generalizations have attracted the attention of the scientific community due to their promising potential for tasks of classification, associative memory, and parallel computation and their ability to solve difficult optimization problems. When a neural circuit is employed as an associative memory, the existence of many equilibrium points is a necessary feature. However, in its application to parallel computation and signal processing involving the solution optimization problems, it is required that system (1) has a unique equilibrium point that is globally attractive. Thus, the global attractivity of systems is of great importance for both practical and theoretical purposes and has been the major concern of most authors dealing with (1) and their generalizations[2~13].

The some results on global asymptotical stability previously quoted concern the case where the neuron activations are assumed bounded and strictly increasing (sigmoidal activations). However, the results of these assumptions are inapplicable to some important engineering problems[8]. In this paper, we relax conditions of activations to Assumption 1 below.

Assumption 1 Function $g_j: \mathbb{R} \to \mathbb{R}$ is globally Lipschitz with Lipschitz constant L_j, i.e., $|g_j(u_j) - g_j(v_j)| \leqslant L_j |u_j - v_j|$ for all $u_j, v_j, j = 1, 2, \cdots, n$.

Our objective in this paper is to study the existence, uniqueness and global stability of the equilibrium point.

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Instead of assuming the boundedness, monotonicity and differentiability of the activation functions and by using M-matrix theory, Lyapunov functions are constructed and employed to establish sufficient conditions for global asymptotic stability.

For convenience, we introduce some notations. $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$ denotes a column vector (the symbol $(^T)$ denotes transpose). $|x| = (|x_1|, \cdots, |x_n|)^T$, $\|x\| = (x_1^2 + \cdots + x_n^2)^{1/2}$. For matrix $A = (a_{ij})_{n\times n}$, A^T denotes the transpose of A, A^{-1} denotes the inverse of A, $[A]^s$ is defined as $[A]^s = (A^T + A)/2$, and $|A|$ denotes absolute-value matrix given by $|A| = (|a_{ij}|)_{n\times n}$, $\|A\| = (\max\{\lambda : \lambda$ is an eigenvalue of $A^TA\})^{1/2}$. If A, B are symmetric matrices, then $A > B$ $(A \geqslant B)$ means that A-B is positive definite (positive semidefinite). $\dot{V}$ denotes time derivative of V.

2 Existence and uniqueness of the equilibrium

Defining the map associated with Eq.(1):

$$H(u) = -Bu + Ag(u) + J. \tag{2}$$

It is known that the equilibria of Eq. (1) are the solutions of the nonlinear equations $H(u) = 0$.

Definition 1 A real matrix $A = (a_{ij})_{n\times n}$ is said to be an M-matrix if $a_{ij} \leqslant 0, i, j, = 1, 2, \cdots, n, i \neq j$, and all successive principal minors of A are positive.

Lemma 1[5,14] Let $A = (a_{ij})_{n\times n}$ be a matrix with non-positive off-diagonal elements. Then it follows that:

1) A is an M-matrix.

2) The real parts of all eigenvalues of A are positive.

3) There exists a vector $\xi > 0$ such that $\xi^T A > 0$.

4) There exists a positive definite diagonal matrix D such that matrix $AD + DA^T$ is positive definite.

5) A is nonsingular and all elements of A^{-1} are nonnegative.

To prove existence and uniqueness of the equilibrium point, we make use of these concepts from topology.

Definition 2 A map $H: \mathbb{R}^n \to \mathbb{R}^n$ is a homeomorphism of $\mathbb{R}^n$ onto itself if $H \in \mathbb{C}^0$, H is one-to-one, H is onto and the inverse map $H^{-1} \in \mathbb{C}^0$.

Lemma 2[8,14] If $H(x) \in \mathbb{C}^0$ satisfies the following conditions:

1) $H(x)$ is injective on $\mathbb{R}^n$,

2) $\|H(x)\| \to \infty$, as $\|x\| \to \infty$;

then $H(x)$ is a homeomorphism of $\mathbb{R}^n$.

Lemma 3 Let $B = \mathrm{diag}(b_1, \cdots, b_n) > 0, L = \mathrm{diag}(L_1, \cdots, L_n) > 0, Q = (q_{ij})_{n\times n}$. If $Q + BL^{-1}$ is an M-matrix, then for each diagonal matrix $K = \mathrm{diag}(k_1, \cdots, k_n)$ satisfying $0 \leqslant K \leqslant L$, we have that $QK + B$ is an M-matrix, and

$$\det(QK + B) \neq 0. \tag{3}$$

Proof Because $Q + BL^{-1}$ is an M-matrix, from Lemma 1, there exist constant numbers $\xi_i > 0$ $(i = 1, 2, \cdots, n)$ such that

$$\xi_j b_j L_j^{-1} + \sum_{i=1}^{n} \xi_i q_{ij} > 0,\ j = 1, 2, \cdots, n. \tag{4}$$

If $k_j > 0$, from Eq.(4) we have

$$\xi_j b_j L_j^{-1} k_j + k_j \sum_{i=1}^{n} \xi_i q_{ij} > 0. \tag{5}$$

Since $b_j > 0$ and $0 < k_j \leqslant L_j$, so $0 < L_j^{-1} k_j \leqslant 1$. From Eq.(5), we get

$$\xi_j b_j + k_j \sum_{i=1}^{n} \xi_i q_{ij} > 0. \tag{6}$$

When $k_j = 0, \xi_j b_j > 0$. So Eq.(6) is true for each k_j such that $0 \leqslant k_j \leqslant L_j (j = 1, 2, \cdots, n)$. From Lemma 1, $QK + B$ is an M-matrix, and Eq.(3) is true. The proof is completed.

Lemma 4 If g satisfies the condition 1), and $\alpha = BL^{-1} - |A|$ is an M-matrix, then for every input J, the map H defined by (2) is injective.

Proof Suppose, for purposes of contradiction, that there exist $x, y \in \mathbb{R}^n$ with $x \neq y$ such that $H(x) = H(y)$. From Eq.(2), we get

$$b_i(x_i - y_i) = \sum_{j=1}^{n} a_{ij}[g_j(x_j) - g_j(y_j)]$$
$$(i = 1, 2, \cdots, n).$$

Taking absolute values for both sides of the above, we obtain

$$b_i |x_i - y_i| \leqslant \sum_{j=1}^{n} |a_{ij}| |g_j(x_j) - g_j(y_j)|. \tag{7}$$

From condition 1), there exist $0 \leqslant k_j \leqslant L_j$ such that $|g_j(x_j) - g_j(y_j)| = k_j |x_j - y_j|$ $(j = 1, 2, \cdots, n)$. So Eq.(7) becomes

$$b_i |x_i - y_i| \leqslant \sum_{j=1}^{n} |a_{ij}| k_j |x_j - y_j|$$
$$(i = 1, 2, \cdots, n). \tag{8}$$

Eq.(8) can be written as matrix from

$$(B - |A| K)|x - y| \leqslant 0.$$

Let $(B - |A| K)|x - y| = c$, so $c \leqslant 0$. From Lemma 3, when $\alpha = BL^{-1} - |A|$ is an M-matrix, $\det(B - |A| K) \neq 0$ for any diagonal matrix K satisfying $0 \leqslant K \leqslant L$. From Lemma 1, all elements of $(B - |A| K)^{-1}$ are nonnegative, so $|x - y| = (B - |A| K)^{-1}c \leqslant 0$, therefore, $x = y$, which is a contradiction. So map H is injective. The proof is completed.

Lemma 5 Suppose g satisfies condition 1), and $\alpha = BL^{-1} - |A|$ be an M-matrix. Then, for every input J, map H is homeomorphism on $\mathbb{R}^n$.

Proof Because $\alpha = BL^{-1} - |A|$ is an M-matrix, from Lemma 4, $H(u)$ is an injective map on $\mathbb{R}^n$. From Lemma 2, if $\|H(u)\| \to +\infty$ as $\|u\| \to +\infty$, then H is homeomorphism on $\mathbb{R}^n$. Let $\bar{H}(u) = -Bu + A(g(u) - g(0))$. To show that H is homeomorphism, it suffices to show that $\bar{H}(u)$ is homeomorphism.

In the following, we will prove that $\|H(u)\| \to \infty$ as $\|u\| \to +\infty$. From the Definition 1, it is easy to verify that $B - |A| L$ is an M-matrix. From Lemma 1, there exists a matrix $C = \mathrm{diag}(c_1, \cdots, c_n) > 0$ such that

$$[C(-B + |A| L)]^s \leqslant -\varepsilon E_n < 0 \qquad (9)$$

for sufficiently small $\varepsilon > 0$. E_n is the identity matrix. Calculating

$$[Cu]^{\mathrm{T}}\bar{H}(u) =$$
$$[Cu]^{\mathrm{T}}[-Bu + A(g(u) - g(0))] \leqslant$$
$$-|u|^{\mathrm{T}}CB|u| + |u^{\mathrm{T}}|C|A||g(u) - g(0)| \leqslant$$
$$-|u|^{\mathrm{T}}CB|u| + |u^{\mathrm{T}}|C|A|L|u| =$$
$$|u|^{\mathrm{T}}[C(-B + |A| L)]^s|u| \leqslant -\varepsilon\|u\|^2.$$

Using Schwartz inequality, we get

$$\varepsilon\|u\|^2 \leqslant \|C\|u\| \|\bar{H}(u)\|,$$

When $\|u\| \neq 0$, we have $\varepsilon\|u\|/\|C\| \leqslant \|\bar{H}(u)\|$. So $\|\bar{H}(u)\| \to +\infty$, i.e. $\|H(u)\| \to +\infty$ as $\|u\| \to +\infty$, From Lemma 2, we know that for every input J, map H is homeomorphism on $\mathbb{R}^n$. The proof is completed.

From Lemma 5, we can obtain the sufficient condition for the existence and uniqueness of the equilibrium in Hopfield neural networks as shown in Theorem 2.

Theorem 1 Suppose g satisfies condition 1), and there exists a positive symmetric matrix P such that $Q = -(B^{\mathrm{T}}P + PB) \leqslant -\mu E_n (\mu > 0)$ is negative definite, and $\delta = \mu - 2L_{\max}\|PA\| > 0$ ($L_{\max} = \max_{1 \leqslant j \leqslant n}\{L_j\}$), then system (1) has a unique equilibrium u^*.

Proof It is only needed to prove that H is homeomorphism on $\mathbb{R}^n$. Firstly, we will prove that H is injective on $\mathbb{R}^n$. Suppose there exist $x, y \in \mathbb{R}^n$ with $x \neq y$ such that $H(x) = H(y)$. From condition 1), there exist $-L_j \leqslant k_j \leqslant L_j$ such that $g_j(x_j) - g_j(y_j) = k_j(x_j - y_j)(j = 1, 2, \cdots, n)$. From Eq.(2), we get

$$b_i(x_i - y_i) = \sum_{j=1}^{n} a_{ij}k_j(x_j - y_j), \quad i = 1, 2, \cdots, n.$$

The above equalities can be written as

$$(-B + AK)(x - y) = 0, \qquad (10)$$

where $K = \mathrm{diag}(k_1, \cdots, k_n)$. Now, we prove indirectly $\det(-B + AK) \neq 0$. Considering the following systems:

$$\dot{z} = (-B + AK)z. \qquad (11)$$

Let $V(z) = z^{\mathrm{T}}Pz$, calculating and estimating the derivative of $V(x)$ along Eq.(11) as follows:

$$\dot{V}(z) = z^{\mathrm{T}}(-BP - PB + 2PAK)z \leqslant$$
$$(-\mu + 2L_{\max}\|PA\|)\|z\|^2 < 0.$$

Now, by means of Lyapunov theorem[14], the trivial solution of Eq.(11) is rendered asymptotically stable, so $\det(-B + AK) \neq 0$ for $-L \leqslant K \leqslant L$. Therefore, from Eq.(10), we get $x = y$, which is a contradiction. So map H is injective.

In the following, we prove that when $\|u\| \to +\infty$, $\|H(u)\| \to +\infty$. Let $\bar{H}(x) = -Bu + A(g(u) - g(0))$. Calculating

$$2u^{\mathrm{T}}P\bar{H} = u^{\mathrm{T}}P\bar{H} + \bar{H}^{\mathrm{T}}Pu =$$
$$-u^{\mathrm{T}}(B^{\mathrm{T}}P + PB)u + 2uPA(g(u) - g(0)) \leqslant$$
$$-\mu\|u\|^2 + 2\|PA\|L_{\max}\|u\|^2 =$$
$$-\delta\|u\|^2.$$

Therefore

$$\delta\|u\|^2 \leqslant 2\|u\|\|P\|\|\bar{H}(u)\|.$$

Simplifying the above inequality, we get

$$\delta\|u\|/(2\|P\|) \leqslant \|\bar{H}(u)\|.$$

So $\|\bar{H}(u)\| \to +\infty$, i.e. $\|H(u)\| \to +\infty$ as $\|u\| \to +\infty$. From Lemma 2, we know that for every input J, map H is homeomorphism on $\mathbb{R}^n$. The proof is completed.

3 Global asymptotic stability of the equilibrium point

Theorem 2 Suppose 1 holds. If $\alpha = BL^{-1} - |A|$ is an M-matrix, then for each $J \in \mathbb{R}^n$, (1) has a unique

equilibrium point, which is globally asymptotically stable.

Proof Since $\alpha = BL^{-1} - |A|$ is an M-matrix, Lemma 5 ensures that H is homeomorphism on $\mathbb{R}^n$. Hence there exists a unique point u^* such that $H(u^*) = 0$, namely, system (1) has a unique equilibrium point u^*. By means of coordinate translation $x(t) = u(t) - u^*$, Eq.(1) can be written as

$$\frac{dx_i(t)}{dt} = -b_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)),\ i = 1,2,\cdots,n, \tag{12}$$

where

$$f_j(x_j) = g_j(x_j + u_j^*) - g_j(u_j^*),$$

$j = 1,2,\cdots,n$. Eqs.(12) have a unique equilibrium at $x = 0$.

Clearly, u^* is globally asymptotically stable for Eq.(1) if and only if the trivial solution of Eq.(12) is global asymptotically stable.

Since $\alpha = BL^{-1} - |A|$ is an M-matrix, from Lemma 1, there exist $\xi_i > 0$ $(i = 1,2,\cdots,n)$ such that

$$-\xi_i b_i + \sum_{j=1}^{n} \xi_j |a_{ji}| L_i < 0,\ i = 1,2,\cdots,n. \tag{13}$$

Consider a Lyapunov functional $V(x)$ defined by $V(x) = \sum_{i=1}^{n} \xi_i |x_i|$. Calculating the upper right derivative $D^+ V$ of V along the solutions of Eq.(12)

$$\begin{aligned} D^+ V(x) &= \\ &\sum_{i=1}^{n} \xi_i \{\operatorname{sgn} x_i [-b_i x_i + \sum_{j=1}^{n} a_{ij} f_j(x_j)]\} \leqslant \\ &\sum_{i=1}^{n} \xi_i \{-b_i |x_i| + \sum_{j=1}^{n} |a_{ij}| |f_j(x_j)|\} \leqslant \\ &\sum_{i=1}^{n} \xi_i \{-b_i |x_i| + \sum_{j=1}^{n} |a_{ij}| L_j |x_j|\} = \\ &\sum_{i=1}^{n} \{-b_i \xi_i + \sum_{i=1}^{n} |a_{ji}| L_i \xi_j\} |x_i| < 0. \end{aligned}$$

By Lyapunov-type Theorem, the trivial solution of Eq.(12) is global asymptotically stable, and therefore, u^* is global asymptotically stable for Eq.(1). The proof is completed.

Corollary For matrix A with $a_{ij} \geqslant 0$, for $i,j = 1, 2,\cdots,n$, suppose condition 1) holds, for every input J, system (1) has a unique equilibrium u^* that is globally asymptotically stable, if and only if $BL^{-1} - A$ is an M-matrix.

Proof From Theorem 2, the sufficient part is true. In the following, we prove the necessary part. Suppose that for every input J, system (1) has a unique equilibrium u^* that is globally asymptotically stable. Then, by means of coordinate translation $x(t) = u(t) - u^*$, Eq.(1) can be written as

$$\frac{dx_i(t)}{dt} = -b_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)),\ i = 1,2,\cdots,n,$$

where

$$f_j(x_j) = g_j(x_j + u_j^*) - g_j(u_j^*),$$

$j = 1,\cdots,n$. The above systems have a unique equilibrium $x = 0$, which is globally asymptotical stable. In special, when $f_j(x_j) = L_j x_j, L_j > 0$ $(j = 1,\cdots,n), x = 0$ is globally asymptotical stable, too. So the matrix $-BL^{-1} + A$ is stable. From definition 1 and Lemma 1, the matrix $BL^{-1} - A$ is an M-matrix. The proof is completed.

Theorem 3 Suppose g satisfies condition 1), and there exists a positive symmetric matrix P such that $Q = -(BP + PB) \leqslant -\mu E_n (\mu > 0)$ is negative definite, and $\delta = \mu - 2L_{\max} \| PA \| > 0$, then for every input J, system (1) has a unique equilibrium u^* that is globally asymptotically stable.

Proof From Theorem 1 for every input J, system (1) has a unique equilibrium u^*. Let $x(t) = u(t) - u^*$, Eq. (1) can be written as

$$\frac{dx(t)}{dt} = -Bx(t) + AF(x), \tag{14}$$

where $F(x) = (f_1(x_1),\cdots,f_n(x_n))^{\mathrm{T}}$ is defined by $F_i(x_i) = g_i(x_i + u_i^*) - g_i(u_i^*)$. Now, consider the Lyapunov function $V(x) = x^{\mathrm{T}} P x$.

$$\begin{aligned} \dot{V} &= -x^{\mathrm{T}}(B^{\mathrm{T}}P + PB)x + 2xPAF(x) \leqslant \\ &\quad -\mu \| x \|^2 \| x \| \ \| PA \| \ \| F(x) \|. \end{aligned}$$

From 1, we obtain $\| F(x) \|^2 \leqslant L_{\max}^2 \| x \|^2$. Thus

$$\dot{V}(x) \leqslant (-\mu + 2 \| PA \| L_{\max}) \| x \|^2 = -\varepsilon \| x \|^2.$$

It follows that the trivial solution of Eq.(14) is globally asymptotically stable, and therefore, u^* is globally asymptotically stable for Eq.(1). The proof is completed.

If the entries of matrix A are nonnegative, Theorem 2 is not only sufficient condition for existence, uniqueness and globally asymptotical stability of the equilibrium for systems (1), but also necessary condition for those. So in this case, Theorem 2 is better than Theorem 3. If

there exist some entries in A are negative, Theorem 2 and Theorem 3 are independent. For example, considering the following systems as follows:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} g_1(u_1) \\ g_2(u_2) \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}. \tag{15}$$

Functions $g_j(v) = \sin(\frac{1}{\sqrt{3}}v) + \frac{1}{\sqrt{3}}v$ satisfy condition 1 wth $L_j = \frac{2}{\sqrt{3}}$ for $j = 1,2$. Then,

$$\alpha = BL^{-1} - |A| = \begin{bmatrix} \frac{\sqrt{3}-1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}-1}{2} \end{bmatrix},$$

but it is not an M-matrix. So Theorem 2 fails. Let $P = E_2$ (E_2 is the identify matrix), $\delta = \mu - 2L_{\max}\|PA\| = 2 - \frac{4}{\sqrt{3}}\frac{1}{2} > 0$. Therefore, by Theorem 3, system (15) has a globally asymptotically stable equilibrium.

4 Conclusion

In the paper, a thorough analysis of existence, uniqueness, and globally asymptotical stability of the equilibrium point for a large class of neural networks has been presented. Without assuming the boundedness, monotonicity and differentiability of the activation functions, the conditions ensuring existence and uniqueness of the equilibrium are obtained. Using M-matrix theory, Lyapunov function are constructed and employed to establish sufficient conditions for global asymptotic stability. The results are applicable to both symmetric and nonsymmetrical interconnection matrices A, and to continuous non-monotonic neuron activation functions.

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