Article ID: 1000 - 8152(2003)02 - 0249 - 04

Adaptive fuzzy modeling for nonlinear systems based on fuzzy competitive learning

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Abstract: The author proposes a new self-tuning fuzzy modeling by means of fuzzy competitive learning. Based on fuzzy competitive learning, the adaptive fuzzy inference is used in fuzzy system. Moreover, based on this modified fuzzy system, the paper presents an on-line identifying algorithm with which the on-line parameter estimation of nonlinear system is realized. To demonstrate the applicability of the proposed method, simulation results are presented at the end of this paper.

Key words: system identification; fuzzy systems; nonlinear system modeling; fuzzy competitive learning; Kalman filtering algorithm

CLC number: TP15　　**Document code**: A

基于模糊竞争学习的非线性系统自适应模糊建模方法

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摘要: 提出了一种新的基于模糊竞争学习的自调整的模糊建模方法.基于模糊竞争学习,模糊系统能够进行自适应模糊推理.在被调整模糊系统基础上,提出了一种非线性系统在线估计参数的在线辨识算法.为了证明提出算法的有效性,最后给出了几个例子的仿真结果.

关键词: 系统辨识; 模糊系统; 非线性系统建模; 模糊竞争学习; 卡尔曼滤波

1 Introduction

As for dynamic systems with complex, ill-conditioned, or nonlinear characteristics, the fuzzy model based on fuzzy sets is a very useful method to describe the properties of dynamic systems using fuzzy inference rules. Many kinds of fuzzy model for modeling and control have been developed since Takagi-Sugeno's model (T-S model) was proposed[1]. Within the fuzzy models stemming from T-S model, the global model output is obtained through the center of gravity defuzzification that is the interpolation of local model outputs. These functional rule models allow us to describe analytically the input-output relation of fuzzy system. However, these fuzzy models have the following problems: those algorithms are complex, the generalization performance of algorithms are bad, and the on-line identification algorithms are not used.

In view of those problems, this paper presents a general method of fuzzy modeling based on fuzzy competitive learning. The competitive learning adopts a principle of learning according to how well it wins[3]. Based on the fuzzy competitive learning, the adaptive fuzzy inference is proposed to identify nonlinear system. Kalman filtering algorithm is used to identify the parameters of conclusion polynomials. To demonstrate the advantages of the proposed method, the paper uses the method to identify nonlinear systems and illustrates the satisfying results in its last part.

2 Description of fuzzy model

In this section, we consider a system $P(U, Y)$ as a multi-input and multi-output system, $U \in \mathbb{R}^m, Y \in \mathbb{R}^q$. For the multi-input and multi-output system, we divide it into q multi-input and single output system. Hence, we only discuss a multi-input and single output system.

Received date: 2000 - 12 - 18; Revised date: 2002 - 09 - 23.

We consider the following format for a multi-input and single output system[6].

R^i: if z is $\bar{z}_i, r_i \mid_{\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_c}$ then

$$y^i = \vec{z}^{\mathrm{T}}\theta_i, \quad i = 1,2,\cdots,c, \tag{1a}$$

$$\bar{y} = \frac{\sum_{i=1}^{c} \mu_i y^i}{\sum_{i=1}^{c} \mu_i} = \sum_{i=1}^{c} \beta_i y^i, \quad \frac{\mu_i}{\sum_{i=1}^{c} \mu_i} = \beta_i,$$

$$\mu_i = \begin{cases} 1 - \dfrac{\| z - \bar{z}_i \|}{r_i}, & \text{if } \| z - \bar{z}_i \| \leqslant r_i, \\ 0, & \text{if } \| z - \bar{z}_i \| > r_i, \end{cases} \tag{1b}$$

where $\mathbb{R}^i$ is the ith rule, z is the input vector, $z = (x_1, x_2, \cdots, x_m)^{\mathrm{T}}$. $\bar{z}_i$ is the ith centroid vector, $\bar{z}_i = (\bar{x}_{i1}, \bar{x}_{i2}, \cdots, \bar{x}_{im})^{\mathrm{T}} (i = 1,2,\cdots,c)$. r_i is the radius of the corresponding input region. y^i is the local output of the ith rule. $\vec{z}$ is the input vector of conclusion part, $\bar{z} = (1, x_1, x_2, \cdots, x_m)^{\mathrm{T}}$. θ_i is the conclusion parameter vector of the ith rule, $\theta_i = (p_{i0}, p_{i1}, \cdots, p_{im})^{\mathrm{T}}$. c is the number of rules. $\bar{y}$ is the output of fuzzy model.

In essence, the local linearization of input and output of system is carried out by means of fuzzy model (1) expressing nonlinear systems. It is obvious that $\| \bar{z} - \bar{z}_i \| \leqslant r_i$ represents the local input region of rules. If any input sample point z belongs to a region or a few regions, satisfying $\| \bar{z} - \bar{z}_i \| \leqslant r_i$, then the union of input regions of rules can cover the input space. Otherwise, the union of input regions of rules does not have to contain the whole input space. In general, if the input region of each rule is determined by the experience of skilled operators or experts according to the input ranges, the union of input regions may equal the whole input space. However, the identifying parameters $\{\bar{z}_i, r_i, \theta_i\}$ corresponding the input region of the each rule are obtained by identification methods. Since the training sample points may not be enough to fill the whole input space or the number of clusters takes a small number, the union of the partitioned input regions may be inferior to the input space of system. As a result, the adaptive fuzzy inference is proposed as follows.

While a new input datum belongs to a region of rules or a few regions of rules, the general fuzzy reasoning is used by means of Eq. (1). On the other hand, while a new input datum does not belong to any region of rules, the adaptive fuzzy inference is proposed through the following steps[6]:

1) Select ada $p_i(0) = r_i (i = 1,2,\cdots,c), l = 1$.

2) ada $p_i(l) =$ ada $p_i(l-1) + \lambda r_i (i = 1,2,\cdots, c)$, if $z \| z(t) - \bar{z}_i \| <$ ada $p_i(l)$, then

$$\mu_i = 1 - \frac{\| z(t) - \bar{z}_i \|}{\text{ada } p_i(l)}, \text{else } \mu_i = 0;$$

3) If $\sum_{i=1}^{c} \mu_i = 0$, then $l = l + 1$, go to 2); if $\sum_{i=1}^{c} \mu_i \neq 0$, then

$$\hat{y} = \frac{\sum_{i=1}^{c} \mu_i (\vec{z}^{\mathrm{T}} \theta_i)}{\sum_{i=1}^{c} \mu_i}.$$

For the above process, the radius of each input region is gradually increased by the growth factor λr_i until the certain region or regions contain the new sample point $z(t)$, satisfying $\| \bar{z} - \bar{z}_i \| < (1 + \sum \lambda) r_i$.

3 On-line identification of fuzzy model

The goal of on-line identification of fuzzy model can on-line update the parameters $\{\bar{z}_i, r_i, \theta_i\}$. The premise parameters including $\bar{z}_i$ and r_i of fuzzy reasoning rules are determined according to the input data. The fuzzy clustering with the fuzzy c-means algorithm (FCM) has been used for generation of the reference fuzzy set[2]. An advantages of this method is that it provides an automatic way of forming of the reference fuzzy sets and does not requires any initial knowledge about the structure in the data set. Unfortunately, the fuzzy clustering is quite time-consuming and may not be suitable for on-line modeling and control. In this paper, based on fuzzy competitive learning to on-line partition fuzzy space of system, an on-line algorithm of fuzzy identification is proposed.

The competitive learning method adopting a principle of learning according to how well it wins is proposed. The goal of competitive learning is to cluser the training patterns into representative groups such that patterns within a cluster are more similar to each than patterns belonging to different clusters. The fuzzy competitive learning algorithm is that every training pattern belongs to a certain degree of every cluster, depending on its distance to the centroid vector. Based on the fuzzy competitive learning algorithm, the fuzzy space structure of system is on-line partitioned.

The fuzzy competitive learning algorithm is shown at follows[3].

1) Select the number of clusters $c(2 \leqslant c \leqslant N)$, and initial centroid vectors $\bar{z}_i(i = 1,2,\cdots,c)$.

2) Determine the membership degree for any sample point,

$$\mu_{ik} = \left[\sum_{j=1}^{c} \left(\frac{\| z(k) - \bar{z}_i \|^2}{\| z(k) - \bar{z}_j \|^2} \right) \right]^{-1}. \tag{2}$$

3) Update centroid vector $\bar{z}_i(i = 1,2,\cdots,c)$,

$$\bar{z}_i(t+1) = \bar{z}_i(t) + \eta(\mu_{ik})^2[z(k) - \bar{z}_i(t)], \tag{3}$$

where η is a leaning constant. $\bar{z}_i$ represents the centroid vector. $z(k)$ represents the sampling value. The fuzzy competitive learning is used to modify the centroid vectors for each training sample point from the above steps. As a result, the calculating time becomes shorter and the convergence speed becomes greater.

The radius r_i of each input region not only determines the size of each input region but also decides the overlapping degree between regions. For this reason, it is important how to select the radius of each input region. To update on-line the radius of each input region, first of all, the overlapping degree λ between region and region is given. Further, the radius of each input region is calculated as follows:

$$r_i = \max_{\substack{j=1,2,\cdots,c \\ i \neq j}} \frac{\| \bar{z}_i - \bar{z}_j \|}{\lambda}, \quad 0 < \lambda < 1. \tag{4}$$

Based on the above process, the fuzzy partitioning space of system and the radius of each input region call be modified on-line.

The output of model (1) is expressed by the following equations:

$$y_k = \sum_{i=1}^{c} \beta_i y_k^i = X(k)\theta, \tag{5}$$

$$X(k) = (\beta_1^k, \cdots, \beta_c^k, \beta_1^k x_1^k, \cdots, \beta_c^k x_1^k, \cdots, \beta_1^k x_m^k, \cdots, \beta_c^k x_m^k), \tag{6}$$

$$\theta = (p_{10}, \cdots, p_{c0}, p_{11}, \cdots, p_{c1}, \cdots, p_{1m}, \cdots, p_{cm})^{\mathrm{T}}, \tag{7}$$

where k represents the kth sampling. To estimate the parameter vector θ, The static Kalman filtering algorithm is used. Here, we apply it to calculate the parameter vector θ as follows[1]:

$$\theta_{k+1} = \theta_k + \frac{S_{k+1} * X_{k+1}^{\mathrm{T}} * (y_{k+1} - X_{k+1} * \theta_k)}{Q + X_{k+1} * S_k * X_{k+1}^{\mathrm{T}}}, \tag{8}$$

$$S_{k+1} = S_k - \frac{S_k * X_{k+1}^{\mathrm{T}} * X_{k+1} * S_k}{Q + X_{k+1} * S_k * X_{k+1}^{\mathrm{T}}}, \quad k = 0,1,\cdots,n-1, \tag{9}$$

where θ is the parameter vector. $\theta_0 = \overline{\text{zero}}$ ($\overline{\text{zero}}$ represents zero vector). $S_0 = \alpha I$ (I is an identity matrix and α is a large positive number). $Q = \exp(-m/N)$ (m represents the iteration counter and N is a constant). y_{k+1} is the real output of the $k+1$ sample point.

4 Simulation examples

Example 1 Consider the gas furnace data of Box and Jenkins as a sample data[4]. This data set is well known and frequently used as a simulated example for test identification algorithms. The data includes 296 couples of real values of input and output. The variable of input is fluid velocity of gas, and the variable of output is the concentration of carbon dioxide. We apply our approach to build fuzzy model based on the Box-Jenkins data set. $y(t-1)$ and $u(t-4)$ are considered as input variables. The sampling interval is 9 s. The number of fuzzy rules is 4. The result is given as follows.

In Table 1, we compare our fuzzy model with other models identified from the same data. It can be seen that the performance of our model is superior to other models

Table 1 Comparison of our model with other models

Name of system model	Input	Number of fuzzy rules	Mean squared error
Tong's model[5]	$Y(t-1), u(t-4)$	19	0.469
Pedryc's model[5]	$Y(t-1), u(t-4)$	81	0.32
Xu's model[5]	$Y(t-1), u(t-4)$	25	0.328
Yoshinari's model[5]	$Y(t-1), u(t-3)$	6	0.299
Our model	$Y(t-1), u(t-4)$	4	0.1007

5 Conclusion

This paper presents a general method of fuzzy modeling based on fuzzy competitive learning. The fuzzy competitive learning is used to modify to centroid vectors for each training sample point. As a result, the input space of fuzzy system can be on-line partitioned. Based on this method, Kalman filtering algorithm is used to identify the parameters of conclusion polynomials. Simulation results demonstrate the calculating time becomes shorter and the convergence becomes faster.

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