Article ID: 1000 - 8152(2002)03 - 0329 - 06

Minimum-Phase/All-Pass Factorization of Nonminimum Phase Systems Using Generalized Interactor Matrix

LIU Yi[1] and HU Ping[2]

(1. Department of Computer Science & Engineering, Southeast University·Nanjing, 210096, P. R. China;
2. College of Information Science and Engineering, Nanjing University of Technology·Nanjing, 210009, P. R. China)

Abstract: This paper is intended to give an explicit expression for minimum-phase/all-pass factorization of any detectable and left invertible multivariable nonminimum phase system. We show that the all-pass part is the inverse of a generalized interactor matrix which corresponds to the unstable invariant zeros of the system. Thus the explicit expression is obtained by directly calculating the generalized interactor matrices. Since our method is a transfer function approach, it can be considered as the complementary to existing state-space approaches.

Key words: generalized interactor matrix; minimum-phase/all-pass; factorization

Document code: A

利用广义内作用矩阵对多变量非最小相位系统进行最小相位/全域通分解

刘 毅[1] 胡 平[2]

(1. 东南大学计算机科学与工程系·南京,210096; 2. 南京工业大学信息科学与工程学院·南京,210009)

摘要: 对于任意可检测和左可逆多变量非最小相位系统,给出了其最小相位/全域通分解的显示解.本文的结果表明全域通因子是系统的广义内作用矩阵的逆矩阵.因此,直接求解系统的广义内作用矩阵便可得到上述频域分解的显示表达式.由于本文的方法是频域法,它可以成为对现有时域法的补充.

关键词: 广义内作用矩阵; 最小相位/全域通; 分解

1 Introduction

It is well known that the minimum-phase/all-pass factorization has played an important role in network and system theory. Since this factorization is dual to the so-called inner-outer factorization, it can also be widely used in H_∞/H_2 optimization[1].

The common features of the existing inner-outer factorization methods[1~6] or the minimum-phase/all-pass factorization methods[7~9] are that the factorizations are expressed in state-space formulas, and either algebraic Riccati equation or Lyapunov equation must be solved. Since the factorization itself is a frequency domain problem, there has arisen a question as to whether the explicit expression can be found by directly manipulating the system transfer matrix. This paper attempts to answer this question and our result is based on the concept of generalized interactor matrix which has been further developed by Mutoh et al[10]. We show that the all-pass part is in fact the inverse of a right generalized interactor matrix which corresponds to the unstable zeros of the system. Thus the explicit expression is found by directly calculating the generalized interactor matrix. Due to the limitation of the space, we only consider the minimum-phase/all-pass factorization of left invertible nonminimum-phase systems which may have infinite zeros or $j\omega$-axis zeros. However, our method can also cope with right invertible systems. A dual result of this paper is the explicit expression for inner-outer (or inner-co-outer) factorization of the systems which may have $j\omega$-axis zeros and infinite zeros. Since our method is a transfer function approach, it can be considered as the complementary to existing state-space approaches.

Throughout the paper, we use $[\cdot]$ and $(\cdot)$ to denote the polynomial matrix and the rational matrix respectively.

Received date: 2001 - 02 - 14; Revised date: 2001 - 10 - 31.

2 Problem statement

Given any detectable and left invertible nonminimum phase system$\Sigma(A,B,C,D)$ with transfer matrix $G(s)$

$$\dot{x} = Ax + Bu,\ y = Cx + D, \qquad (2.1a)$$

$$G(s) = C(sI - A)^{-1}B + D, \qquad (2.1b)$$

where $x \in \mathbb{R}^n, y \in \mathbb{R}^p, u \in \mathbb{R}^m$, and B, C are full rank. The purpose of this paper is to find an explicit expression for the following minimum-phase-image/all-pass factorization of the system:

$$G(s) = G_m(s)V(s),\ V(s)V^{\mathrm{T}}(-s) = I, \qquad (2.2)$$

where $G_m(s)$ is called the minimum-phase-image of Σ, and can be realized as $\Sigma_m(A, B_m, C, D_m)$. $V(s)$ is square, stable, biproper and all pass.

3 Nilpotent interactor and generalized interactor

Due to the definition of left nilpotent interactor matrix[11], a right nilpotent interactor matrix is defined as follows

Definition 3.1 For any proper and full rank transfer matrix $\hat{G}(z) \in \mathbb{R}^{p\times m}$, any polynomial matrix $\hat{L}[z] \in \mathbb{R}^{m\times m}$ with the following properties will be called a right nilpotent interactor matrix of $\hat{G}(z)$.

$$\lim_{z\to\infty}\hat{G}(z)\hat{L}[z] = M, \qquad (3.1)$$

where M is a full rank real matrix, and

$$\det(\hat{L}[z]) = cz^k, \qquad (3.2)$$

where c is a constant.

A right nilpotent interactor matrix can be found using the duality of the algorithm proposed in [12]. The definition of the nilpotent interactor matrix implies the following lemma[12].

Lemma 3.1 For any proper and left invertible transfer matrix $\hat{G}(z) \in \mathbb{R}^{p\times m}$ with right nilpotent interactor matrix $\hat{L}[z]$,

$$\deg\{\det(\hat{L}[z])\} = \rho, \qquad (3.3)$$

where ρ is the total number of infinite zeros of $\hat{G}(z)$, or the lowest relative degree of $m \times m$ minors of $\hat{G}(z)$.

Like the definition of the interactor matrix which extracts infinite zeros from $G(s)$, it is also possible to define a generalized interactor matrix which extracts all the invariant unstable zeros and jω zeros from $G(s)$[11]. In the following, we will give the definition of a right generalized interactor matrix.

Definition 3.2 For the system (2.1) with the set of jω-axis zeros and unstable zeros $\{s_1, s_2, \cdots, s_d\}$ not necessarily distinct, any biproper matrix $L(s) \in \mathbb{R}^{m\times m}$ with the following properties will be called a right GIM (generalized interactor matrix) of $G(s)$ (comparing with the definitions made in [11], the generalized interactor matrix defined here does not extract the infinite zeros of $G(s)$).

1) Satisfying

$$\lim_{s\to s_i} G(s)L(s) = \Lambda_i,\ i = 1,\cdots,d, \qquad (3.4)$$

where Λ_i is a full rank real matrix.

2) The zeros of $L(s)$ are in LHP.

3) The poles of $L(s)$ are $s_1, s_2, \cdots, s_d$.

To show how to calculate a GIM of $G(s)$, we first assume that $G(s)$ have only one invariant zero at $\mathrm{Re}(s) \geqslant 0$. Then substituting $z = \dfrac{s+\alpha}{s-s_1}$ into $G(s)$ will transform the zero at $s = s_1$ to a zero at $z = \infty$, so that finding a GIM of $G(s)$ is equal to finding a nilpotent interactor matrix of the transformed system. When $G(s)$ has more than one distinct zeros at $\mathrm{Re}(s) \geqslant 0$, let us denote the set of distinct jω-axis zeros and unstable zeros of $G(s)$ as $\{s_1, s_2, \cdots, s_{\bar{d}}\}$. Just as in the single zero case, we can obtain

Lemma 3.2 $L(s)$ can be calculated by the following algorithm[11].

Step 1 Use

$$z = \frac{s+\alpha_1}{s-s_1},\ \alpha_1 > 0 \qquad (3.5)$$

to transform $G(s)$ into $\hat{G}(z)$, and compute the right nilpotent interactor of $\hat{G}(z)$ denoted as $\hat{L}_1[z]$. Then use (3.5) again to transform $\hat{L}_1[z]$ back to $L_1(s)$, and calculate

$$G_1(s) \triangleq G(s)L_1(s). \qquad (3.6)$$

Step i Use

$$z = \frac{s+\alpha_i}{s-s_i},\ \alpha_i > 0 \qquad (3.7)$$

to transform $G_{i-1}(s)$ into $\hat{G}_{i-1}(z)$ $(G(s) \triangleq G_0(s), \hat{G}_0(z) \triangleq \hat{G}(z))$, and compute the right nilpotent interactor of $\hat{G}_{i-1}(z)$ denoted as $\hat{L}_i[z]$. Then use (3.7) again to transform $\hat{L}_i[z]$ back to $L_i(s)$, and calculate

$$G_i(s) \triangleq G_{i-1}(s)L_i(s). \qquad (3.8)$$

Repeating the above step until $i = \bar{d}$, then we have

$$L(s) = L_1(s)L_2(s)\cdots L_{\bar{d}}(s), \qquad (3.9a)$$

$$G_{\bar d}(s) \triangleq G_{\bar d-1}(s)L_{\bar d}(s) = G(s)L(s). \qquad (3.9b)$$

Remark 3.1 Since the nilpotent polynomial matix $\hat{L}_i[z]$, $i = 1,\cdots,\bar d$ satisfies (3.2), the transform (3.7) implies that all the poles and zeros of $L_i(s)$ are at s_i and $-\alpha_i$ respectively, and the algebraic multiplicity of s_i as a pole of $L_i(s)$ is equal to the algebraic multiplicity of $-\alpha_i$ as a zero of $L_i(s)$[14]. Moreover, from Lemma 3.1, it follows that the algebraic multiplicity of s_i as a pole of $L_i(s)$ is equal to the algebraic multiplicity of s_i as an unstable zero (or jω-axis zero) of $G(s)$.

Remark 3.2 (3.9b) shows that all the unstable zeros of $G(s)$ are exactly canceled by all the poles of $L(s)$, thus $G_{\bar d}(s)$ and $G(s)$ have the same pole structure (In the case when pole-zero cancellation happens, $G_{\bar d}(s)$ can still be realized with the same order as Σ). Except that $G(s)$ has zeros at s_i while $G_{\bar d}(s)$ has zeros at $-\alpha_i$ and the algebraic multiplicity of s_i as a zero of $G(s)$ is equal to the algebraic multiplicity of $-\alpha_i$ as a zero of $G_{\bar d}(s)$, the remaining invariant zero structures of $G(s)$ and $G_{\bar d}(s)$ are the same. $G_{\bar d}(s)$ is minimum phase. Moreover, since $L(s)$ is biproper, the infinite zero structures of $G(s)$ and $G_{\bar d}(s)$ are the same and $G_{\bar d}(s)$ is left invertible. Therefore the generalized right interactor matrix $L(s)$ can be seen as a transform which only maps the unstable invariant zeros of $G(s)$ to the LHP without affecting other pole-zero structures of $G(s)$.

Remark 3.3 The zeros of $G_{\bar d}(s)$, $-\alpha_1,\cdots,-\alpha_d$, which correspond to the unstable zeros of $G(s)$ can be at arbitrary place in LHP.

Definition 3.3 The $L(s)$ in Definition 3.2 will be called a right unitary GIM, if

$$L(s)L^{\mathrm{T}}(-s) = I_m. \qquad (3.10)$$

Usually it is difficult to find a unitary GIM. However, the problem can be simplified if we choose $\alpha_i = s_i$ in (3.7).

Definition 3.4 The nilpotent interactor $\hat{L}_i[z]$ in Lemma 3.2 will be called a right unitary interactor, if

$$\hat{L}_i[z]\hat{L}_i^{\mathrm{T}}[z^{-1}] = I_m. \qquad (3.11)$$

Lemma 3.3 Assuming that no invariant zeros of Σ lie on the jω-axis, if in Lemma 3.2, $L_i(s)$, $i = 1,\cdots,\bar d$, is calculated by choosing $\alpha_i = s_i$ in (3.7), then a sufficient condition for $L(s)$ to be a unitary GIM is that $\hat{L}_i[z]$ is a unitary interactor.

Proof It follows from $z = \dfrac{s+s_i}{s-s_i}$ and $L_i(s) = \hat{L}_i[z]$ that

$$L_i(-s) = \hat{L}_i\left[\frac{-s+s_i}{-s-s_i}\right] = \hat{L}_i[z^{-1}],$$

so that

$$L_i(s)L_i^{\mathrm{T}}(-s) = \hat{L}_i[z]\hat{L}_i^{\mathrm{T}}[z^{-1}] = I_m,\ i = 1,\cdots,\bar d.$$

Therefore $L(s)$ is a unitary GIM.

Lemma 3.3 converts the problem of finding a unitary GIM to the problem of finding unitary interactors. As shown in [13], for any proper and full rank transfer matrix $\hat{G}(z)$, its left unitary interactor can be obtained by using the same algorithm used for calculating nilpotent interactor matrix[12]. Here what we need is the right unitary interactor, and, this can be achieved by using the duality between left and right interactor matrix.

Definition 3.5 The $m\times m$ first degree polynomial matrix

$$U^{(j)}[z] = \begin{bmatrix} 0 & zI_{k_j} \\ I_r & 0 \end{bmatrix},\ m = r + k_j \qquad (3.12)$$

will be called a column shift polynomial matrix (CSPM) of order k_j.

Lemma 3.4 For each $\hat{G}_{i-1}(z)$, $i = 1,\cdots,\bar d$ in Lemma 3.2, there exists a right unitary interactor matrix $\hat{L}_i[z]$ consisting of finite t_i factors

$$\hat{L}_i[z] = S_i^{(1)}[z]S_i^{(2)}[z]\cdots S_i^{(t_i)}[z], \qquad (3.13a)$$

$$S_i^{(j)}[z] \triangleq Q_i^{(j)}U_i^{(j)}[z],\ j = 1,\cdots,t_i, \qquad (3.13b)$$

where $U_i^{(j)}[z]$ is a CSPM of order k_j and $Q_i^{(j)}$ is a $m\times m$ unitary real matrix. $U_i^{(j)}[z]$ and $Q_i^{(j)}$ can be calculated using the duality of the algorithm proposed in [12].

Proof By using the duality, (3.13) can be proved in the same way as in [12]. The proof consists of two parts: first, we present an algorithm to calculate the unitary interactor. Second, we show that the number of factors in (3.13b) is finite.

Part 1 The transfer matrix $\hat{G}_{i-1}(z)$ can always be factored as

$$\hat{G}_{i-1}(z) = \bar{D}^{-1}[z]\bar{N}[z], \qquad (3.14)$$

where $\bar{D}[z]$ and $\bar{N}[z]$ are polynomial matrices which are in the form of

$$\bar{D}[z] \triangleq Iz^{\bar n} + \bar{D}_1 z^{\bar n-1} + \cdots + \bar{D}_{\bar n}, \qquad (3.15a)$$

$$\overline{N}[z] \triangleq \overline{N}_0 z^{\bar{n}} + \overline{N}_1 z^{\bar{n}-1} + \cdots + \overline{N}_{\bar{n}} \qquad (3.15b)$$

for some $\bar{n}$ and may not be coprime. Since the denominator $\overline{D}[z]$ in (3.15) is a monic polynomial matrix, the nilpotent interactor matrix $\hat{L}_i[z]$ can be evaluated from the numerator polynomial $\overline{N}[z]$, i.e.,

$$\begin{aligned}&\lim_{z\to\infty}\overline{D}^{-1}[z]\overline{N}[z]\hat{L}_i[z] = \\ &\lim_{z\to\infty}\overline{D}^{-1}[z]z^{\bar{n}}\lim_{z\to\infty}z^{-\bar{n}}\overline{N}[z]\hat{L}_i[z] = \\ &\lim_{z\to\infty}z^{-\bar{n}}\overline{N}[z]\hat{L}_i[z].\end{aligned} \qquad (3.16)$$

The algorithm is started by setting $\overline{N}^{(0)}[z] = \overline{N}[z]$ and $\hat{L}_i^{(0)}[z] = I_m$. Consider lth iteration in calculating $\hat{L}_i[z]$ using (3.13).

Step 1 If $r_l = \operatorname{rank}\overline{N}_0^{(l-1)} = m$, where $\overline{N}_0^{(l-1)}$ denotes the coefficient matrix of $z^{\bar{n}}$ in $\overline{N}^{(l-1)}[z]$, then the algorithm terminates and the unitary interactor is $\hat{L}_i[z] = \hat{L}_i^{(l-1)}[z]$, and $t_i = l - 1$.

If $r_l < m$, factor $\overline{N}_0^{(l-1)}$ into following QR decomposition form

$$\overline{N}_0^{(l-1)} = [0_l \quad \overline{N}_{0D}^{(l)}](Q_i^{(l)})^{-1}, \text{ i.e.,} \qquad (3.17a)$$

$$\overline{N}_0^{(l-1)}Q_i^{(l)} = [0_l \quad \overline{N}_{0D}^{(l)}], \qquad (3.17b)$$

where $Q_i^{(l)}$ is an $m \times m$ unitary real matrix, $k_l = m - r_l$, and 0_l is a k_l-column zero matrix.

Step 2 Premultiply $\overline{N}^{(l-1)}[z]$ by matrix $Q_i^{(l)}$ and denote it as $\hat{N}[z]$,

$$\hat{N}[z] = \overline{N}^{(l-1)}[z]Q_i^{(l)}. \qquad (3.18)$$

Now, the leading coefficient of $\hat{N}[z]$ is equal to the right-hand side of (3.17b).

Step 3 Premultiply $\hat{N}[z]$ by a CSPM of order k_l and set

$$\overline{N}^{(l)}[z] = \hat{N}[z]U_i^{(l)}[z]. \qquad (3.19)$$

This multiplication shifts coefficient matrix of $z^{\bar{n}}$ in $\hat{N}[z]$ forward by k_l columns. Let

$$\hat{L}_i^{(l)}[z] = \hat{L}_i^{(l-1)}[z]S_i^{(l)}[z]. \qquad (3.20)$$

And this ends the ith iteration.

Combining (3.17b) ~ (3.20), we have

$$\begin{aligned}&\overline{N}^{(l)}[z] = \\ &\overline{N}^{(l-1)}[z]Q_i^{(l)}U_i^{(l)}[z] = \\ &\overline{N}^{(l-1)}[z]S_i^{(l)}[z] = \overline{N}[z]\hat{L}_i^{(l)}[z],\end{aligned} \qquad (3.21)$$

where $S_i^{(l)}[z]$ and $\hat{L}_i^{(l)}[z]$ are defined by (3.13b) and (3.20).

The final iteration yields

$$\overline{N}^{(t_i)}[z] = \overline{N}[z]\hat{L}_i^{(t_i)}[z], \qquad (3.22)$$

where $\hat{L}_i^{(t_i)}[z] = \hat{L}_i[z]$ which is defined by (3.13a). Since $\operatorname{rank}\overline{N}_0^{(t_i)} = m$, property (3.1) is satisfied. Noting that $\det(U_i^{(j)}[z]) = z^{k_j}$ and $\det(Q_i^{(j)}) = 1$, we have property (3.2). Moreover, since $U_i^{(j)}[z]\{U_i^{(j)}[z^{-1}]\}^{\mathrm{T}} = I$ and $Q_i^{(j)}$ is a unitary real matrix, it follows that $\hat{L}_i[z]$ is a unitary interactor.

Part 2 Since $\hat{G}_{i-1}(z)$ is left invertible, it follows from Lemma 3.1 that

$$\deg\{\det(\hat{L}_i[z])\} = \sum_{j=1}^{t_i} k_j = \rho_i, \qquad (3.23)$$

where ρ_i is the algebraic multiplicity of the unstable zero s_i. Hence, t_i must be a finite number.

4 An explicit expression

For the minimum-phase/all-pass factorization, the results of Lemma 3.2 ~ 3.4 lead to the following theorem.

Theorem 4.1 For the system $\Sigma(A,B,C,D)$ as in (2.1) which may have infinite zeros but does not have $j\omega$-axis zeros, let $L(s)$ be a right unitary GIM of Σ which is calculated by using Lemma 3.2 ~ 3.4, then

1) The minimum-phase/all-pass factorization is expressed as

$$G(s) = G_m(s)V(s), \quad V(s)V^{\mathrm{T}}(-s) = I_m, \qquad (4.1)$$

where

$$G_m(s) = G_{\bar{d}}(s), \quad V(s) = L^{-1}(s). \qquad (4.2)$$

2) $G_m(s)$ can be realized as $\Sigma_m(A, B_m, C, D_m)$.

Proof 1) We note that all the zeros and poles of $L(s)$ are $-s_1, \cdots, -s_d$ and $s_1, \cdots, s_d$ respectively, it follows from (3.9b) that $G_{\bar{d}}(s)$ is a minimum phase image of $G(s)$. Equation (3.9b) can be rewritten as

$$G(s) = G_{\bar{d}}(s)L^{-1}(s). \qquad (4.3)$$

$L(s)$ being a unitary GIM implies that $L^{-1}(s)$ is all-pass.

2) $G(s)$ can be factored as

$$G(s) = D_p^{-1}[s]N_p[s], \qquad (4.4)$$

where $N_p[s]$, $D_p[s]$ are polynomial matrix, and $D_p[s]$ is row reduced. Combining (4.3) and (4.4)

$$G_{\bar{d}}(s) = D_p^{-1}[s]N_p[s]L(s). \qquad (4.5)$$

Since all the poles of $L(s)$ is the unstable zeros of $N_p[s]$, it follows that $\hat{N}_p[s] \triangleq N_p[s]L(s)$ is a polynomial matrix. Thus

$$G_{\bar{d}}(s) = D_p^{-1}[s]\hat{N}_p[s] \qquad (4.6)$$

is a left MFD (matrix fraction description) of $G_{\bar{d}}(s)$. Comparing (4.6) with (4.4), and using the method for observer-form realizations from left MFDs[14], it follows that $G_{\bar{d}}(s)$ can be realized as $\Sigma_m(A, B_m, C, D_m)$.

D_m and B_m are found using the following method. $G_m(s)$ can be expressed as

$$G_m(s) = G_m'(s) + D_m, \tag{4.7}$$

where $G_m'(s)$ is strictly proper. Suppose the observability matrix $\Phi(A, C)$ has rank $r \leqslant n$, we can find a similarity transformation matrix T to transform $\Sigma(A, B, C, D)$ into the following observable/unobservable form[14]

$$\bar{A} = T^{-1}AT,\ \bar{B} = T^{-1}B,\ \bar{C} = CT, \tag{4.8a}$$

$$\bar{A} = \begin{array}{c} r \\ n-r \end{array}\begin{pmatrix} \bar{A}_o & 0 \\ \bar{A}_{21} & \bar{A}_{\bar{o}} \end{pmatrix},\ \bar{B} = \begin{pmatrix} \bar{B}_0 \\ \bar{B}_{\bar{o}} \end{pmatrix},\ \bar{C} = (\bar{C}_o \quad 0), \tag{4.8b}$$

where $\{\bar{C}_o, \bar{A}_o\}$ is observable. Then B_m may take the form of

$$B_m = T\begin{pmatrix} \hat{B}_o \\ 0 \end{pmatrix}, \tag{4.9}$$

where $\hat{B}_o$ satisfies

$$\bar{C}_o(sI - \bar{A}_o)^{-1}\hat{B}_o = G'_m(s). \tag{4.10}$$

Equation (4.10) implies that

$$\sum_{i=1}^{\infty} \bar{C}_o\bar{A}_o^{i-1}\hat{B}_o s^{-i} = \sum_{i=1}^{\infty} h_i s^{-i}, \tag{4.11}$$

where h_i is the Markov parameter of $G_m'(s)$. Let

$$\bar{\Phi}^{\mathrm{T}} = [\bar{C}_o^{\mathrm{T}}, \bar{A}_o^{\mathrm{T}}\bar{C}_o^{\mathrm{T}}, \cdots, (\bar{A}_o^{\mathrm{T}})^{r-1}\bar{C}_o^{\mathrm{T}}]. \tag{4.12}$$

Therefore,

$$\hat{B}_o = (\bar{\Phi}^{\mathrm{T}}\bar{\Phi})^{-1}\bar{\Phi}^{\mathrm{T}}[h_1^{\mathrm{T}}, h_2^{\mathrm{T}}, \cdots, h_r^{\mathrm{T}}]^{\mathrm{T}}. \tag{4.13}$$

Remark 4.1 Since the discovery of a unitary GIM is based on finding unitary nilpotent interactor matrices, the complexity of our method is largely dependent on the algorithm to calculate the unitary nilpotent interactor matrices. However, the algorithm proposed in [12] for calculating the nilpotent interactor matrix is simple and amenable to computer-based calculations because it only carries out QR decomposition on $p \times m$ real matrices. Moreover, since our method does not need to solve the Riccati equation, it can be expected to have good numerical reliability than existing state-space approaches.

5 Conclusions

This paper has provided an explicit expression for the minimum-phase/all-pass factorization of any detectable and left invertible nonminimum phase system which may have infinite zeros or jω zeros. The novelty of our method is that the all pass part is explicitly expressed as the inverse of a generalized right interactor matrix of the system. The result can also be extended to detectable and right invertible systems. A dual result of this paper is the inner-outer (or inner-co-outer) factorization of any stabilizable nonminimum-phase system which is either left invertible or right invertible and may have infinite zeros or jω zeros (to this end, we only need to exclude the jω-axis zeros form the set $\{s_1, \cdots, s_{\bar{d}}\}$). Since the explicit expression for minimum-phase/all-pass factorization is found without solving the Riccati equation, our method is more explicit and can have better numerical reliability than existing state-space approaches.

References

[1] Doyle J C. Lecture notes in advances in multivariable control [R]. Minneapolis, USA: ONR/Honeywell Workshop, 1984

[2] Chen Tand Francis B A. Spectral and inner-outer factorizations of rational matrices [J]. SIAM J. on Matrix Analysis and Applications, 1989, 10(1): 1 – 17

[3] Yeh F and Wei L. Inner-outer factorizations of right-invertible real-rational matrices [J]. Systems & Control Letters, 1990, 14(1): 31 – 36

[4] Tsai M C and Chen L W. A numerical algorithm for inner-outer factorizations of real-rational matrices [J]. Systems & Control Letters, 1993, 20(3): 209 – 217

[5] Hara S. Inner-outer factorization for strictly proper functions with jω-axis zeros [J]. Systems & Control Letters, 1991, 16(2): 179 – 185

[6] Copeland B R and Safonov M G. Zero cancelling compensations for singular control problems and their application to the inner-outer factorization problem [J]. Int. J. Robust Nonlinear Control, 1992, 2 (2): 139 – 164

[7] Shaked U. An explicit expression for the minimum-phase image of transfer function matrices [J]. IEEE Trans. Automatic Control, 1989, 34(12): 1290 – 1293

[8] Chen B M, Saberi A and Sannui P. Explicit expressions for cascade factorization of general nonminimum phase systems [J]. IEEE Trans. Automatic Control, 1992, 37(3): 358 – 363

[9] Lin Z, Chen B M and Saberi A. Explicit expressions for cascade factorizations of general non-strictly proper systems [J]. Control Theory and Advanced Technology, 1993, 9(4): 501 – 515 (in Japanese)

[10] Mutoh Y. Generalized interactor matrix and model matching for linear multivariable nonminimum phase systems [J]. Trans. of the Society of Instrument and Control Engineers, 1994, 30(1): 24 – 30 (in Japanese)

[11] Rogjozinski M W, Paplinski A P and Gibbard M J. An algorithm

for the calculation of a nilpotent interactor matrix for linear multivariable systems [J]. IEEE Trans. Automatic Control, 1987, 32 (3): 234 - 237 (in Japanese)
[12] Mutoh Y. Interactor matrix and structure at infinity in linear multivariable control systems [J]. J. of the Society of Instrument and Control Engineers, 1993, 32(3): 237 - 252 (in Japanese)
[13] Peng Y and Kinnaert M. Explicit solution to the singular LQ regulation problem [J]. IEEE Trans. Automatic Control, 1992, 37(5): 633 - 636
[14] Kailath T. Linear Systems [M]. Englewood Cliffs, NJ: Prentice-Hall, 1980

本文作者简介

刘　毅　1963年生.1985年和1988年毕业于东南大学自动控制系,分别获得学士和硕士学位.1997年获得日本冈山大学自然科学研究科人工智能专业工学博士学位.现就职于东南大学计算机系.主要研究方向是多变量自适应控制和非线性控制. Email: riuyi@163.com

胡　平　1962年生.副教授.1986年毕业于东南大学获得学士学位,2001年毕业于南京工业大学(原南京化工大学)获得硕士学位.目前主要从事计算机优化控制,计算智能等方面的研究工作. Email: huping@njuct.edu.cn

Main Contents of the Next Issue