Normal mode fluctuation of sound propagating in random shallow water

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Abstract: Research of fluctuation of sound propagation in a random shallow water environment is of great interest in predicting sound field and environmental parameters for random ocean investigation and monitoring. This paper studies the normal mode fluctuation of sound propagation in random shallow water. Monte Carlo simulation is used to analyze the sound field fluctuation induced by sound speeds random fluctuation with Dozier's statistical coupled mode equations in a random ocean. The random sound speed model is derived based on the statistic analysis of a temperature chain data in shallow sea. The results show that the amplitude fluctuation of normal mode induced by sound speed random fluctuation is larger than the amplitude fluctuation of sound pressure. If the source position is close to the sensitive depth of a normal mode, the scintillation index of this normal mode would be 2~3 order of magnitude larger than the variance of normal mode amplitude fluctuation. As such, the scintillation index of normal mode is an important parameter for detecting sound propagating fluctuation. This article reveals that variance of the sound pressure fluctuation is linear against distance, variance of normal mode amplitude fluctuation, and modal scintillation index.

Key words: random fluctuation; normal mode; shallow water

随机浅海声传播简正波起伏

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摘要: 随机浅海环境中声传播起伏的研究,对于随机海洋环境声场预报及海洋环境参数反演和遥测都有十分重要的意义。文中对随机浅海中声传播的简正波起伏进行了研究,利用 L.B.Dozier 统计耦合模式方程,通过 Monte Carlo 数值模拟分析声速场随机变化所产生的声场起伏规律。随机声速场模型由海上温度链测量数据统计规律分析得到。结果表明,声速场随机起伏所导致的简正波幅度起伏远高于声压的起伏;当声源处在某一阶简正波的敏感深度位置时,该阶简正波的闪烁系数会比简正波幅度起伏方差高 2~3 个量级;在所分析随机声速场模型下,声压起伏方差、简正波幅度起伏方差及简正波闪烁系数随距离的增加基本上是线性增大。

关键词: 随机起伏;简正波;浅海

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1 INTRODUCTION

The parameters of ocean environment are temporal and spatial variables. Fluctuation of sound propagation in random ocean is always an important subject in underwater acoustics. The main factors that lead to the ocean random inhomogeneity include rough surface, bottom roughness and the random changes of temperature, salinity and density. The inhomogeneity of these factors can cause sound scattering, and changes in the spatial distribution of sound energy and amplitude or phase of sound signal which effect the detection and identification of sound signal. The random inhomogeneity of sea water is caused mainly by random internal wave, turbulence and meso-scale eddy which are much more intensive at the surface. Therefore, the sound field fluctuation caused by the inhomogeneity of sea water is more apparent at the shallow areas. The investigation of sound propagating fluctuation in random heterogeneous ocean[2-5] has been conducted in many cases both in terms of theories and experiments. Shallow water normal mode consists of much information regarding the various ocean environment parameters. Therefore, the variation of parameters can be extracted[6,7]. By detection and analysis the normal mode of shallow water. The relation between variation of the normal mode propagation and the variation of environment parameters needs to be identified. This paper addresses the problem based on the research of Dozier's statistics random normal mode theory[1], and the regularity of normal mode fluctuation. The advantage of Dozier's method is that the random sound field is expressed in terms of a superposition of the local normal mode of the mean sound speeds, therefore the statistical singularity of the projection of the sound field on the normal mode can be obtained. The average ocean environment can be obtained in advance, such that a relatively stable background can be achieved for the detection of the random environment. The results of numerical simulation show that fluctuation of normal mode coefficient, the amplitude of normal mode, which is based on the mean sound speeds, is correlated with statistical regularity of sound speed field, and the fluctuation of each normal mode and the source depth. If the source depth is close to the sensitive depth of some modes, the fluctuation of this mode of amplitude is relatively greater. This phenomenon can be expressed by the modal scintillation index. The scintillation of modal energies is often used to characterize and understand acoustics wave propagation in a randomly fluctuating ocean waveguide[8]. V. Premus[9] used the modal scintillation index to express the fluctuation of normal mode amplitude excited by random fluctuation of the source depth, and further to monitoring and classifying the sources in shallow water. He proposed a variant of the traditional modal scintillation index to treat the discrimination problem in a typical littoral oceanic waveguide. The approach is based on a modal decomposition of fluctuation in the received pressure field associated with the temporal modulation of the depth of an acoustic source about its mean value. The variation regularity of modal scintillation index excited by the variation of random environment has been considered, and the position of the source was counted at the same time. The result of analyzing the fluctuation regularity of normal mode in shallow water shows that modal scintillation index is an important parameter in analyzing the sound field fluctuation in shallow water.

Section 2 describes the data observation and analysis by a temperature chain over a period of 33 hours in South China Sea and the establishment of an ocean environment model for numerical simulation model based on the observed results. Section 3 introduces briefly the Dozier's statistics random normal mode theory and the definition of modal scintillation index. Section 4 describes the numerical simulation methodology and the analysis of the results of numerical simulation. A summary of the results is highlighted in Section 5.

2 DATA COLLECTION AND ANALYSIS

Fig.1 shows the measurements of water temperature using a temperature chain in South China Sea. The temperature chain is 190m long and contains 29 units. In monitoring the changes of temperature, density and salinity of seawater, the variation of temperature is the most sensitive factors in the sound propagation. Therefore, the monitoring process was focused at the temperature variation. Fig.2 shows the sound speed data transformed from Fig.1 by the following empirical equation[10]

$$c=1449.2+4.6T-0.055T^2+0.00029T^3+(1.34-0.01T)(S-35)+0.016Z$$

where c, s and z represent sound speed, salinity and depth.

Fig.2 shows the sound speed fluctuation against time. Fig.3 shows the PDF (probability density function) of sound speeds in Fig.2 at 80m and 160m water depth, which is close to Gauss distribution. The variance of sound speeds at different depths is given in Fig.4 denoted solid curved line. Fig.4 shows the sound speed fluctuation in three divisions along its depth. The first section (5m-25m) shows the least sound speed variation. The second section (25m-95m) shows the most sound speed fluctuation. The sound speed variation in the third section (95m-190m) is a little less than that in the second section.

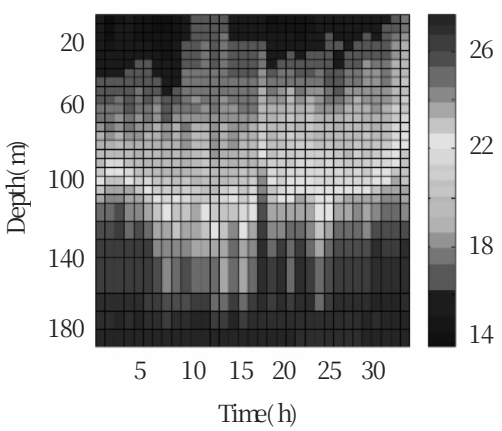


Fig.1 Temperature data measured by the temperature chain

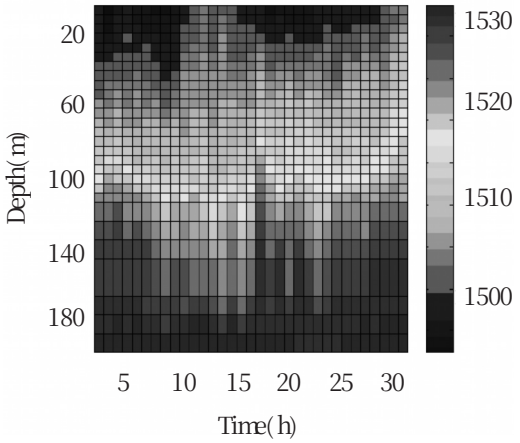


Fig.2 Sound speed data

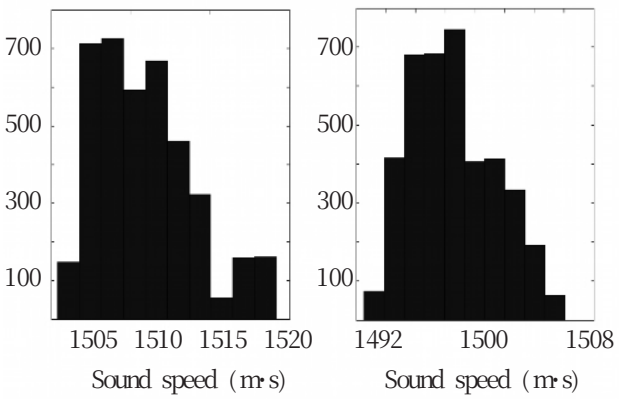


Fig.3 PDF of sound speed at 80m and 160m water depth

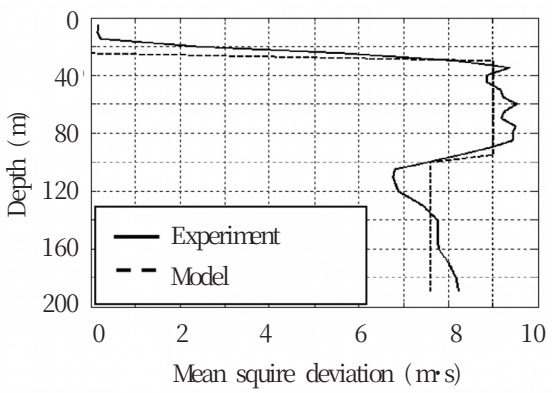
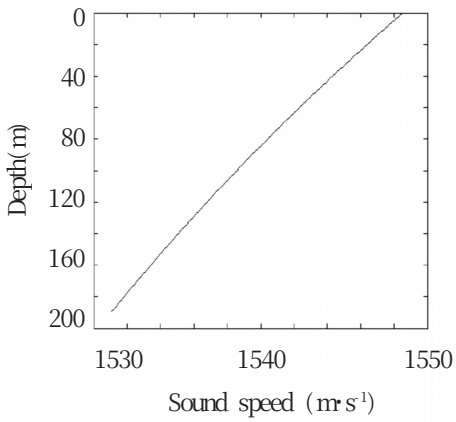


Fig.4 Variance of the sound speed fluctuation

Based on the experimental results, a random sound speed field model for the numerical simulation was established. Using a Pekris waveguide with depth 190m, water density of 1000kg/m^3, bottom sound speed at 2000m/s, bottom density of 2000 kg/m^3, source depth of 100m bellow the surface and frequency of 100Hz, the sound speed in water was modeled as $c(r, z, t)=\bar{c}+\delta c(r, z, t)$. Where the background sound speed is $\bar{c}$ and δc is the random sound speed fluctuation which is a random variable.

The background sound speed is assumed to have a negative gradient of sound speed profile as shown in Fig.5(a). The random fluctuation δc is assumed to have a random variable which confirms with Gauss distribution and zero mean value. According to the above measurements, the district modal of δc is shown Fig.5(b). The variance of sound speed fluctuation δc over the three sections mentioned are 0m/s (0-25m), 9m/s (25-95m), and 7.6m/s (95-195m). The variance curve is shown as a dashed line in Fig.4. The background speed is the mean speed of the random sound speeds. Monte Carlo numerical simulation was applied in this ocean environment model for the analysis of the regularity of normal mode fluctuation induced by the random fluctuation of sound speeds.



(a) Background sound speed profile

(b) Variance of sound speed field schematic of the waveguide

Fig.5 Sound speed profile model

3 THEORY

3.1 Dozier's equation

The statistical coupled normal mode equation for low frequency modelling was presented by Dozier[7] with quasi-static assumption. The advantage of this method is that the sound field fluctuation is expressed in the form of fluctuation projection on the normal mode of the mean sound speeds, i.e. expressing the sound field in terms of normal mode of mean sound speeds. The fluctuation of sound field induced by the random fluctuation of the environment can be illustrated by the fluctuation of its projection on the normal mode of the mean sound speeds. Therefore, it is possible to monitor and inverse the random fluctuation of environment parameters by the actual measurements. The local normal mode of the mean sound field in a specific sea area could be obtained as the prior data for analysis. Such prior data can be used to express the sound field with the superposition of normal mode. The variation of the normal mode amplitude (or coefficient) caused by fluctuation of environment parameters can be obtained. From the variation of the normal mode coefficient, it is possible to extract the information of the ocean environment parameters fluctuation. The monitoring of ocean environment by means of acoustical method is possible.

In the waveguide introduced above, sound field of simple harmonic point source at r=0 in cylindrical coordinates satisfies the boundary conditions and the equation below

$$\frac{\partial^2 p}{\partial t^2} - c^2(r,z,t)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{\partial^2 p}{\partial z^2}\right]=0 \qquad (1)$$

in the far field

$$\frac{\partial^2 P}{\partial t^2} - c^2\left[\frac{\partial^2 P}{\partial r^2}+\frac{\partial^2 P}{\partial z^2}\right]=0$$

where $P=\sqrt{r}\,p$

assuming the mean sound field is distance independent, $\bar{c}(r,z,t)=\bar{c}(z,t)$, the local normal mode ϕ_n of the mean sound speeds in z direction satisfies the boundary conditions and the equation

$$\frac{d^2\phi_n}{\partial z^2}+\frac{\omega^2}{\bar{c}^2(z)}\phi_n=l_n^2\phi_n \qquad n=1,2,\cdots,N$$

where l_n is the normal mode wave number and ω is frequency of sound source, yielding

$$P(r,z,t)=\exp(-j\omega t)\sum_{n=1}^{N}A_n(r,t)\phi_n(z)$$

Assuming perturbation $|\delta c/c| \ll 1$, adopting quasistatic approximation and ignoring the back scattering, the normal mode coefficient An satisfies the differential equation[7]

$$\frac{\partial \Psi_n(r,t)}{\partial r} = -j\sum R_{nm}(r,t)\exp(jl_{nm}r)\,\Psi_m(r,t) \qquad 1 \le n \le N \quad (2)$$

$$R_{nm}(r,t) = \frac{\omega^2}{c_0^2(l_n l_m)^{\frac{1}{2}}}\int dz \frac{\delta c(r,z,t)}{c_0}\phi_n(z)\phi_m(z)$$

where, $\xi_n = \sqrt{l_n}A_n$, $\Psi_n = \exp(-jl_n r)\xi_n$, $l_{nm} = l_{mn}$, Eq.(2) is the Dozier's statistical coupled normal mode equation. If the initial condition $\Psi_n(0,t) = \Psi^0_n(t)$ is known, we can get A_n at range r. One set $A_n(r,t)$ can be get for one sound speed sample. By Monte Carlo numerical method we can get the statistical distributions of A_n in the random ocean environment.

3.2 Scintillation index of normal mode

Scintillation index of normal mode introduced in reference[9] is used here. In the far field of an acoustic source, the complex pressure field at a depth z may be expressed in terms of a superposition of normal modes, ϕ_n, given by

$$P(z,r,z_s) = \alpha\sum_{m=1}^{M}\phi_m(z_s)\phi_m(z)\frac{\exp(jk_m r)}{\sqrt{k_m r}}$$

where z_s is source depth, k_m is the horizontal wave number corresponding to mode $\phi_m(z)$, and a is a complex, zero-mean random variable. In vector notation, temporal samples of the noise added pressure field p(t), may be written as

$$p(t) = \phi H(t) + n(t),$$

where ϕ is the N×M matrix of normal mode functions, H(t) is the M×1 vector of temporally varying modal excitation, $\alpha\phi_m(z_s(t))[\exp(jk_m r)/\sqrt{k_m r}]$, n(t) is an N×1 noise vector, and N is the number of sensors.

The scintillation index, SI_m, for each mode is defined as

$$SI_m = \frac{Var\{|\hat{h}_m(t)|\}}{E\{|\hat{h}_m(t)|\}} \qquad (3)$$

where $\hat{h}_m(t)$ represents the mth element of $\hat{H}_m(t)$, $\hat{H}_m(t)$ is the estimate of H(t), and $|\cdot|$ denotes magnitude. In this paper, $\hat{H}_m(t)$ is the result of sound speeds variation stochastic. It is obtained by Mounte Carlo simulation, then the modal scintillation index, SI_m, can be calculated using Eq.(3).

4 NUMERICAL SIMULATION

Monte Carlo numerical simulation was carried out for the ocean model mentioned in Section 2. We sampled the sound speed randomness in 2×2 spatial mesh, and took 100 samples. Fig.6 shows the PDF of δc at 80m and 160m, with the corresponding variances of 11.487 and 5.8614. The adopted source depth was 100m below the surface with a frequency at 100Hz. Fig.7 shows the normal mode of the background environment calculated by running program KRAKAN[11], and a total of 7 modes was examined. Fig.8 and Fig.9 show the PDF of A_n at distance 1 km and 3 km from source calculated by Eq.(2). Tab.1 shows the absolute value of normal mode coefficient in the mean sound speeds. Tab.2 shows the variances of the total sound pressure at different distances. Tab.3 and Tab.4 show the variances of normal mode coefficient and the modal scintillation

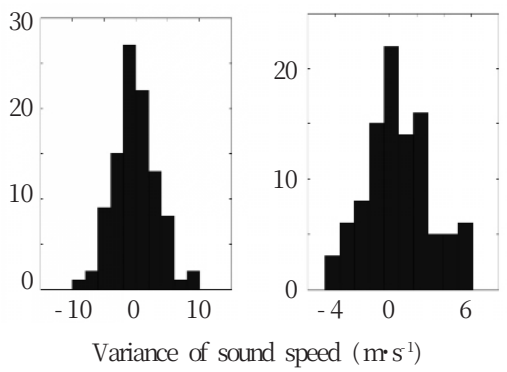


Fig.6 PDF of sound speed at 80m and160m

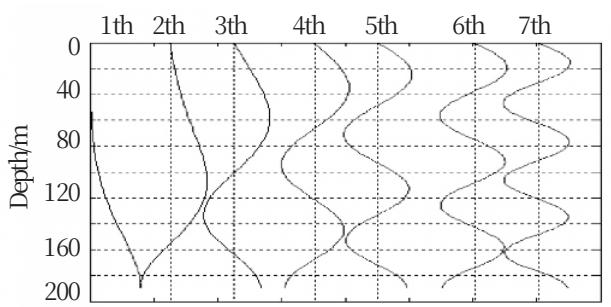


Fig.7 Normal mode of the background

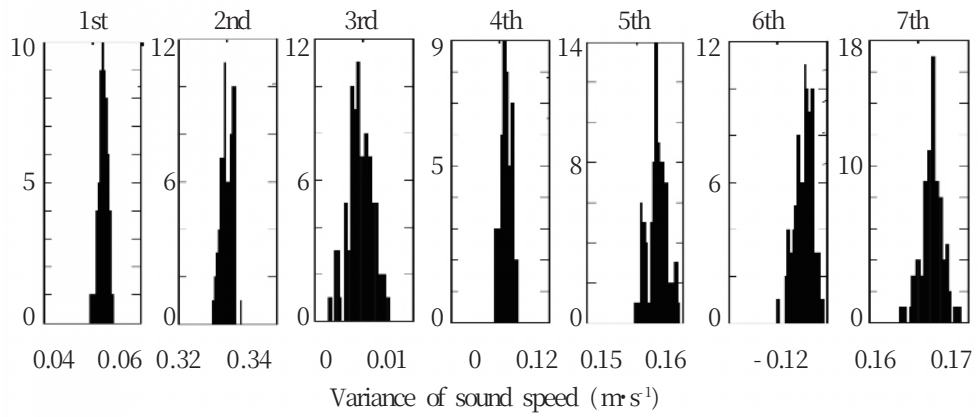


Fig.8 PDF of An at 1 km

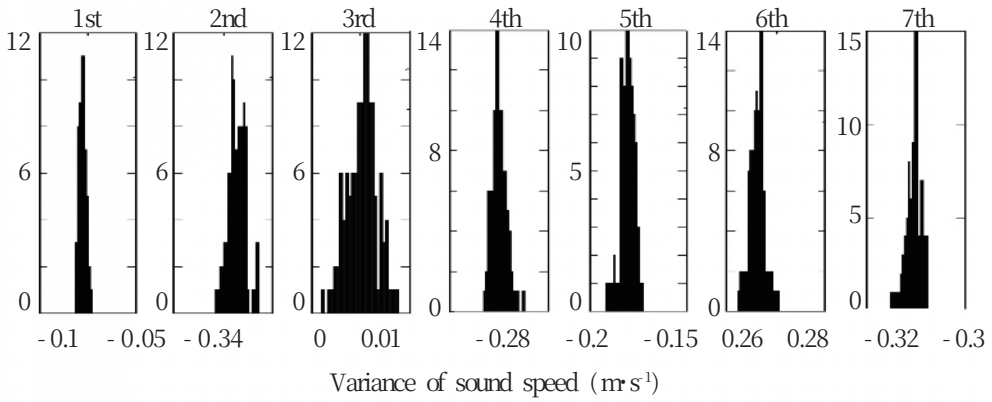


Fig.9 PDF of An at 3 km

Table 1 Coefficients of normal mode in mean sound speed field

	1st mode	2nd mode	3 rd mode	4th mode	5th mode	6th mode	7th mode
$\lvert An \rvert$	0.1082	0.4578	0.0019	0.3625	0.2296	0.2545	0.3209

Table 2 Variance of sound pressure at different distances and depths

$\times 10^{-7}$	20m	40m	60m	80m	100m	120m	140m	160m	180m
1km	1.1304	0.3626	0.8419	1.0646	1.1693	4.8263	1.2498	1.0155	0.9948
3km	3.8125	2.9275	3.5307	3.9683	2.5352	2.0186	2.1350	3.8566	5.4902
5km	5.9424	3.7014	5.1910	5.6739	2.5954	3.5955	6.6514	5.8110	6.3579

Table 3 Variance of normal mode coefficient

$\times 10^{-4}$	1st mode	2nd mode	3 rd mode	4th mode	5th mode	6th mode	7th mode
1km	0.0415	0.0299	0.0538	0.0207	0.0383	0.0281	0.0295
3km	0.1189	0.0787	0.1539	0.0554	0.1111	0.0917	0.0831
5km	0.1772	0.1221	0.2023	0.1074	0.1495	0.1580	0.1573

Table 4 Modal scintillation index

$\times 10^{-3}$	1st mode	2nd mode	3 rd mode	4th mode	5th mode	6th mode	7th mode
1km	0.0383	0.0050	0.9659	0.0057	0.0166	0.0110	0.0091
3km	0.1097	0.0171	1.9406	0.0152	0.0438	0.0360	0.0259
5km	0.1638	0.0266	1.6634	0.0296	0.0651	0.0621	0.0490

index. Fig.8 and Fig.9 show that the normal mode coefficients fluctuate randomly when the sound speeds fluctuate, and the fluctuation of the normal mode coefficients follow the Gauss distribution. It can be seen from Tab.2 that the total pressure fluctuates too as the sound speed field fluctuating. The change of magnitude for the variance of the sound pressure amplitude ranges from 10^{-8}~10^{-7}. Tab.4 and Tab.5 show the magnitude changes of variance within the normal mode coefficient fluctuation from 10^{-6}~10^{-5} over the range from 1km to 5km, and that the corresponding normal mode scintillation index is in the range of 10^{-6}~10^{-3}. Amongst the parameters of sound pressure, normal mode coefficient and modal scintillation index, normal mode coefficient fluctuation is larger than the sound pressure fluctuation caused by fluctuation of the sound speed field, and the fluctuation of modal scintillation index is the most sensitive one. Therefore, it is more favorable to monitor and inverse the

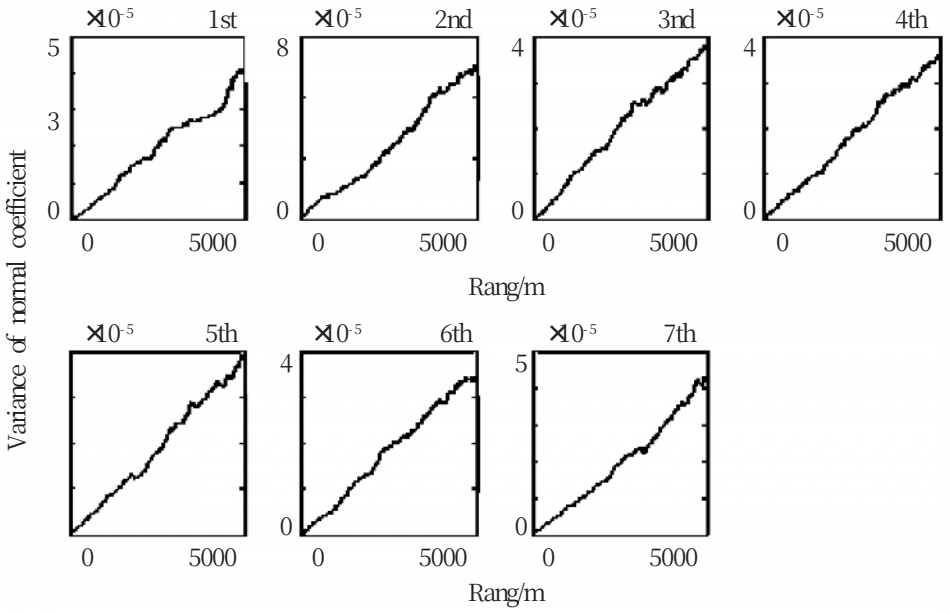


Fig.10 Variance of normal mode coefficient versus distance

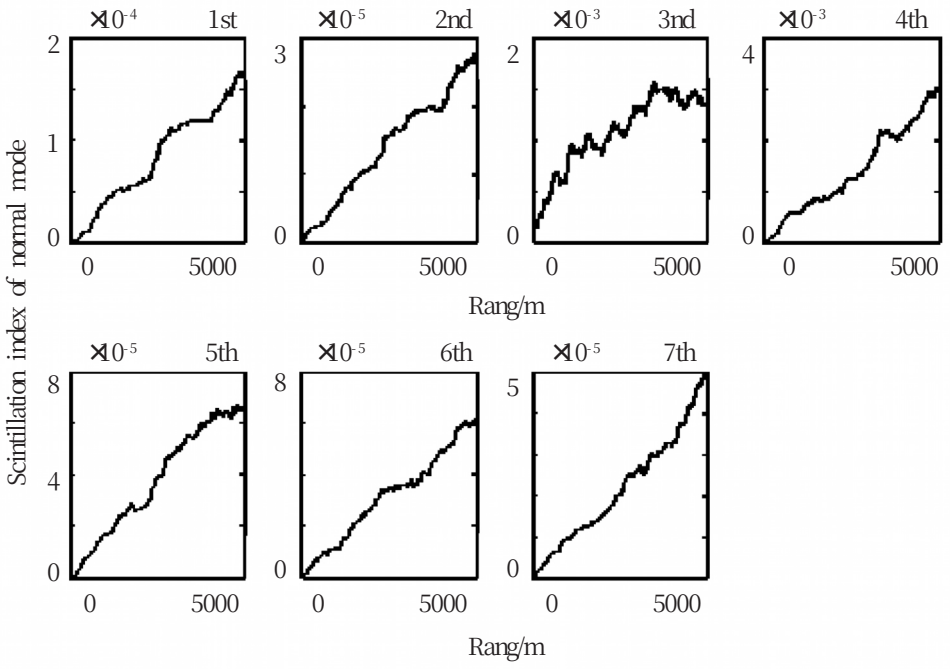


Fig.11 Scintillation index of normal mode versus distance

ocean environment parameters by coefficient or scintillation index of normal mode.

It can be seen from Tab.3 that the variance of the normal mode coefficient fluctuation is in the same order for each mode. However, as shown in Tab.4, the modal scintillation index of the 3rd mode is 1-2 order of magnitude larger than the others. The reason is that the source position is just right for the wave node of the 3rd mode (see Fig.7). Premus's study shows that when the source position is just right for the wave node, the normal mode amplitude of this mode is sensitive to the source position and the PDF of the corresponding modal scintillation index is higher. Therefore, the source position can be classified by statistic characteristics of all modal scintillation indices. It is also found from the study that when the source position is just right for the wave node, the modal scintill-ation index is sensitive to random fluctuation of the sound speeds, the modal scintillation index can increase by 2 order of magnitude. The phenomenon is valuable for acoustic monitor in shallow water.

Fig.10 and Fig.11 show the variance of normal mode coefficient and the modal scintillation index versus distance. These Figures show that the variance of normal mode coefficient and the modal scintillation index is linear with distance. Only the scintillation index of 3 rd mode which is obviously larger than the others presents saturated tendency at 3.5km.

5 SUMMARY

This paper studied the fluctuation of normal mode induced by random fluctuation of sound speeds in the shallow water. The model of random fluctuating sound speeds in shallow water is based on the temperature data observed using a temperature chain. Dozier's statistical random normal mode theory has been adopted to proceed with the numerical analysis. Numerical simulation was conducted using Monte Carlo experimental method. The results show that fluctuation of normal mode coefficient is far larger than the sound field fluctuation. If the source position is close to the sensitive depth of some exponential normal mode, the scintillation index of this normal mode is in the 2-3 order of magnitude larger than the variance of the modal coefficient fluctuation. Therefore, the scintillation index of normal mode is an important parameter for detecting of sound propagating fluctuation. The model indicates that the variance of normal mode coefficient is linear with distance as well as modal scintillation index.

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